Mathematical Logic: Proof theory, Constructive Mathematics

Organised by
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Abstract. The workshop “Mathematical Logic: Proof Theory, Constructive Mathematics” was centered around proof-theoretic aspects of current mathematics, constructive mathematics and logical aspects of computational complexity.

Mathematics Subject Classification (2010): 03Fxx.

Introduction by the Organisers

The workshop Mathematical Logic: Proof Theory, Constructive Mathematics was held November 16-22, 2014 and included two tutorials:

(1) Thierry Coquand: Univalent Foundation and Constructive Mathematics (2 times 1 hour),
(2) Ulrich Kohlenbach, Daniel Körnlein, Angeliki Koutsoukou-Argyraki, Laurenţiu Leuştean: Proof-Theoretic Methods in Nonlinear Analysis (2 times 50 min plus 2 times 30).

Coquand’s tutorial gave a general introduction on the univalent foundation program of Voevodsky and discussed the construction of the cubical set model of type theory in a constructive metatheory. This model satisfies the computation rules for equality introduced by P. Martin-Löf as judgemental equality.

The second tutorial developed the proof-theoretic framework for the unwinding of proofs in nonlinear analysis and outlined recent applications to: image recovery problems (Part I, Kohlenbach), fixed point theory of pseudocontractive mappings...
(Part II, Körnlein), convex optimization (Part III, Leuştean) and abstract Cauchy-problems given by accretive operators (Part IV, Koutsoukou-Argyraki).

In addition to these tutorials, 29 talks of mostly 25 minutes were given aiming:

To promote the interaction of proof theory and computability theory with core areas of mathematics as well as computer science via the use of proof interpretations. J. Avigad’s talk studied the amount of algorithmic randomness needed in Weyl’s theorem on uniform distributions. H. Towsner showed how to arrive at Tao’s version of Szemerédi’s regularity lemma as the functional interpretation of a measure-theoretic \( \Pi^0_3 \)-statement. H. Schwichtenberg reported on a machine extracted program from the Nash-Williams minimal bad sequence argument for Higman’s lemma. V. Brattka introduced a concept of Las Vegas computable functions to calibrate the computational power of randomized computations on real numbers. A. Weiermann described a general formula for the computation of the maximal order types for well quasi orders arising in the combinatorics of finite multisets. P. Schuster showed how a reformulation of transfinite methods in algebra as admissible rules can be used to eliminate uses of such methods from proofs of sufficiently simple statements in abstract algebra. On the side of applications to concrete applications in computer science, M. Seisenberger reported on applications of logic to the verification of railway control systems and U. Berger developed a proposal to optimize programs extracted by proof-theoretic methods to be able to e.g. control their complexity, allow for partial data and to override data that are no longer used.

To further develop foundational aspects of proof theory and constructive mathematics. S. Artemov talked on intuitionistic epistemic logic which is based on the BHK-semantics and treats intuitionistic knowledge as the result of a verification. F. Aschieri reported on a new proof-theoretic method to extract Herbrand disjunctions from classical first-order natural deduction proofs. B. Afshari’s talk also studied Herbrand’s theorem, this time in terms of certain tree grammars assigned to proofs of existential statements in first-order logic. The talk by G.E. Leigh addressed the issue of cut-elimination for first-order theories of truth. P. Oliva presented new results on a game-theoretic interpretation of Spector’s bar recursion, a more efficient novel variant of bar recursion and recent uses in the analysis of the Podelski-Rybalchenko termination theorem. F. Ferreira showed how a suitable functional interpretation can be used to give an ordinal analysis of Kripke-Platek set theory. B. van den Berg reported on new developments in the functional interpretation of systems of nonstandard analysis. T. Streicher talked on models of classical realizability (in the sense of J.-L. Krivine) arising from domain-theoretic models of \( \lambda \)-calculus with control. The talks by L.D. Beklemishev and J.J. Joosten addressed recent progress in the area of provability logic with applications to ordinal analysis. Also on the side of ordinal analysis was a talk by T. Strahm, who developed a so-called flexible type system in the spirit of S. Feferman whose strength is measured by the small Veblen ordinal. S. Berardi presented
a new rule-learning based approach to the proof-theoretic analysis of second order arithmetic. A. Bauer talked about constructive homotopy theory and models of intensional type theory. I. Petrakis proposed a formalization of so-called Bishop spaces as a constructive foundation for point-function topology. A. Swan studied the existence property for intuitionistic set theories where this property has to be understood in terms of definability. M. Rathjen reported on his recent proof of a conjecture due to Feferman which states that the continuum hypothesis CH is not definite in the technical sense that a certain semi-intuitionistic set theory does not prove CH ∨ ¬CH.

To explore further the connections between logic and computational complexity. Talks in this area spanned the topics of propositional proof complexity, set-theoretic computation, and complexity theoretic aspects of bounded arithmetic. P. Pudlák reported on work-in-progress and new conjectures for two propositional proof systems based on integer linear programming, the cutting planes proof system and the Lovász-Schrijver proof system. N. Thapen reported new results about size and width tradeoffs for propositional resolution refutations, including new lower bounds via the colored PLS (polynomial local search) principle. S. Buss presented a new framework of polynomial-time computation for set functions based on Cobham-style limited recursion using ∈-recursion. A. Beckmann described a proof-theoretic analysis for the polynomial-time computable set functions based on safe/normal ∈-recursion. L. Kołodziejczyk discussed recent progress on complexity-theoretic aspects of the Paris-Wilkie problem on the relationship between bounded arithmetic, the (negation) of exponentiation, and collection.

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Workshop: Mathematical Logic: Proof theory, Constructive Mathematics

Table of Contents

Sergei N. Artemov
   Provability vs. computational semantics for intuitionistic logic ........ 2941

Vasco Brattka (joint with Guido Gherardi and Rupert Hölzl)
   Probabilistic Choice and Las Vegas Computability ...................... 2942

Thomas Strahm (joint with Florian Ranzi)
   A flexible type system for the small Veblen ordinal .................... 2943

Helmut Schwichtenberg (joint with Monika Seisenberger)
   Higman’s lemma and its computational content ......................... 2943

Jeremy Avigad
   Uniform distribution and algorithmic randomness ......................... 2944

Lev D. Beklemishev
   Recent results on provability algebras ................................. 2945

Leszek Kołodziejczyk
   The Paris-Wilkie problem of the consistency of no collection and no exponentiation ........................................... 2946

Ulrich Kohlenbach
   Proof-theoretic methods in nonlinear analysis I: Logical Foundations and Some Applications ........................................... 2947

Daniel Körnlein
   Proof-theoretic methods in nonlinear analysis II: Fixed Point Theory ... 2948

Graham E. Leigh
   Eliminating cuts in theories of truth ................................... 2949

Federico Aschieri (joint with Margherita Zorzi)
   Some recent results on Herbrand’s Theorem ................................ 2951

Sam Buss (joint with A. Beckmann, S.D. Friedman, M. Müller, N. Thapen)
   Cobham Recursive Set Functions ........................................... 2952

Peter Schuster (joint with Davide Rinaldi)
   Transfinite Methods as Admissible Rules ................................... 2953

Monika Seisenberger (joint with Andrew Lawrence, Ulrich Berger, Phil James, Fredrik Nordvall-Forsberg, and Markus Roggenbach)
   Applications of Logic to the Verification of Railway Control Systems 2954
Thierry Coquand
Univalent Foundation and Constructive Mathematics .................. 2955

Paulo Oliva
Recent Applications of Bar Recursion and Selection Functions ....... 2956

Iosif Petrakis
Bishop spaces: constructive point-function topology .................... 2958

Andreas Weiermann (joint with Michael Rathjen, Jeroen Van der Meeren)
Well quasi orders .................................................................................. 2960

Laurențiu Leuştean (joint with Ulrich Kohlenbach, Adriana Nicolae)
Proof-theoretic methods in nonlinear analysis III: Quantitative results on Fejér monotone sequences .................................................. 2960

Angeliki Koutsoukou-Argyraki (joint with Ulrich Kohlenbach)
Proof-theoretic methods in nonlinear analysis IV: Rates of convergence and metastability for abstract Cauchy problems generated by accretive operators ........................................................................... 2961

Bahareh Afshari (joint with Stefan Hetzl and Graham E. Leigh)
Grammars for first-order proofs ......................................................... 2963

Joost J. Joosten
Ordinal analysis based on Turing progressions .............................. 2964

Pavel Pudlák
On proof systems for integer linear programing ............................. 2966

Neil Thapen
A trade-off between length and width in resolution ....................... 2967

Henry Towsner
Finitary and Infinitary Approaches to Szemerédi Regularity ............... 2969

Stefano Berardi
A rule-learning based interpretation for second order arithmetic (Stefano Berardi, Torino University) ............................................................. 2970

Andrej Bauer
Constructive homotopy theory and models of intensional type theory .... 2971

Arnold Beckmann (joint with Sam Buss, Sy-David Friedman, Moritz Müller, and Neil Thapen)
Proof Theoretic Characterisations of Feasible Set Functions ............... 2972

Thomas Streicher
Classical Realizability arising from Domain Theoretic Models of Lambda Calculus with Control ................................................................. 2973

Benno van den Berg (joint with Eyvind Briseid and Pavol Safarík)
Results around a nonstandard functional interpretation .................. 2974
Andrew Swan
  *Definability and Non-Definability in Intuitionistic Logic* .......... 2975

Fernando Ferreira
  *A functional interpretation of KPω* ................................. 2976

Ulrich Berger
  *Logical representations of partial, mutable and reusable data* ........ 2977

Michael Rathjen
  *CH and semi-intuitionism* ........................................... 2980