

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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**Mathematics in Undergraduate Study Programs:
Challenges for Research and for the Dialogue between
Mathematics and Didactics of Mathematics**

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ABSTRACT. The topic of undergraduate mathematics is of considerable concern for mathematicians in universities, but also for those teaching mathematics as part of undergraduate studies other than mathematics, for employers seeking to employ a mathematically skilled workforce, and for teacher education. Different countries have made and continue to make massive efforts to improve the quality of mathematics education across all age ranges, with most of the research undertaken particularly at the school level. A growing number of mathematicians and mathematics educators now see the need for undertaking interdisciplinary research and collaborative reflections around issues at the tertiary level. The conference aimed to share research results and experiences as a background to establishing a scientific community of mathematicians and mathematics educators whose concern is the theoretical reflection, the research-based empirical investigation, and the exchange of best-practice examples of mathematics education at the tertiary level. The focus of the conference was mathematics education for mathematics, engineering and economy majors and for future mathematics teachers.

Mathematics Subject Classification (2010): Primary: 97C70; Secondary: 97D60, 97D30, 97D40, 97D70, 97C60.

Introduction by the Organisers

The workshop *Mathematics in Undergraduate Study Programs: Challenges for Research and for the Dialogue between Mathematics and Didactics of Mathematics*, focused on *Mathematics education at the tertiary level* and organised by Rolf Biehler (Institut für Mathematik, Universität Paderborn), Reinhard Hochmuth (Institut für Mathematik und ihre Didaktik, Leibniz Universität Hannover), Dame Celia Hoyles (London Knowledge Lab, UCL Institute of Education, University of London) and Patrick W. Thompson (Dept. of Mathematics and Statistics, Arizona State University), took place at Oberwolfach, Dec 7-13th 2014. The topic of undergraduate mathematics is of considerable concern for mathematicians in universities, but also for those teaching mathematics as part of undergraduate studies other than mathematics, for employers seeking to employ a mathematically skilled workforce, and for teacher education. Different countries have made and continue to make massive efforts to improve the quality of mathematics education across all age ranges, with most of the research undertaken particularly at the school level. A growing number of mathematicians and mathematics educators now see the need for undertaking interdisciplinary research and collaborative reflections around issues at the tertiary level. The conference aimed to share research results and experiences as a background to establishing a scientific community of mathematicians and mathematics educators whose concern is the theoretical reflection, the research-based empirical investigation, and the exchange of best-practice examples of mathematics education at the tertiary level. The focus of the conference was mathematics education for mathematics, engineering and economy majors and for future mathematics teachers.

Aims were: (1) To create and sustain a *national and international interdisciplinary community of mathematicians and researchers in mathematics education* who are interested in tertiary level mathematics education. (2) To share and discuss a range of topics and concerns with a clear focus on *grounded approaches in theoretical and empirical aspects*, thus moving the field beyond the simple exchange of opinions and beliefs.

The workshop set out to share results and best practice around the following topics:

- The transition between school and university mathematics
- The content and goals of undergraduate and first year university mathematics education given new developments in mathematics at school level and at university level
- The design and evaluation of competence-oriented curricula elements including innovative methods for assessing students knowledgetaking into account needs of the specific study programmes (mathematics majors, teachers, engineers)
- Didactical analyses of mathematical content including general aspects such as formal representations, proving, and specific aspects concerning the learning of calculus and linear algebra

- Knowledge about subject specific learning obstacles and learning strategies, the development of motivation and beliefs, development of instruments that take the specificity of mathematics into account
- The design and evaluation of support systems for students including tutoring, mentoring and student support centres
- Difficulties that university mathematics departments face, both cognitive and cultural, in attempts to enhance the quality of students mathematical thinking and understanding

The conference was organised around the following sections: Learning mathematics at the undergraduate level, mathematical meaning making, transition between school and university, technology, reasoning and proof, maths in teacher education, maths as a service subject at university in particular in engineering studies, and common ground for research in undergraduate mathematics teaching. Each theme included keynote talks followed by small group discussions chaired by different participants, who made notes of the main issues raised and shared them with the whole group in a plenary session at the end of the conference. The conference brought together experts from eight countries, in particular from Great Britain, France, USA and Germany.

Reflections and next steps

Aim 1: There was much discussion on the nature of the school/university transition in general, with particular reference to topics that are known to present challenges for students such as understanding limits and real numbers. Several themes became very apparent and were taken up in small group discussions: how to promote the shift from intuitive to rigorous reasoning, from pragmatic to conceptual approaches, or from a naive to a scientific point of view. Discussion centred on how this might be managed in ways that were intellectually honest but also practical. Regarding intellectual honesty, one approach put forward was that didactics could usefully be considered as ‘applied mathematics’, where mathematics is actually seen to be used and fundamental in the solution of problems or there is an *intellectual necessity* for a mathematical point of view.

Aim 2: Many presentations addressed methodological challenges for research in tertiary mathematics education, not least in terms of the adequacy of theoretical frameworks and the need for mathematicians and mathematics educators to work together on common projects. Two fruitful ways forward were suggested by some presentations: one related to analysing student responses to carefully constructed assessment questions and conjecturing about the routes of any misconceptions; another using interdisciplinary design research methodologies with the aim to address known didactical or epistemological obstacles through detailed task designs and evaluations while exploiting digital technologies in ways that enable students to explore the relevant mathematical structures, ways simply not possible with paper and pencil tools.

The workshop was remarkable in that it highlighted the diversity of undergraduate students mathematical experiences among countries. It seems there must be

greater clarity of what is *meant* by mathematics teaching at university level before we can consider if and how we might change it.

There was an overriding consensus that it is the collective responsibility of mathematicians and mathematics educators to work together on common projects to make progress and take the field forward.

The workshop ended with real and shared motivation to engage further, and in particular work together in more collaborative research. There are projects to further the conference's presented work: (1) a special issue of IJRUME (International Journal of Research in Undergraduate Mathematics Education) with original research papers from the workshop will be published, and (2) shared ways to devise a coherent agenda of systematic research to produce results robust enough to transcend country divisions (e.g. the International Network for Didactic Research in University Mathematics initiative).

The challenges are not to be underestimated. To achieve these there is a need for methodological advances as well as further opportunities to share, perhaps at the khdm follow up conference "Didactics of Mathematics in Higher Education as a Scientific Discipline" to be held in December 2015 in Hannover at Schloss Herrenhausen.

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**Workshop: Mathematics in Undergraduate Study Programs:
Challenges for Research and for the Dialogue between Mathe-
matics and Didactics of Mathematics**

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