Real Analysis, Harmonic Analysis and Applications

Organised by
Michael Christ, Berkeley
Detlef Müller, Kiel
Christoph Thiele, Bonn

20 July – 26 July 2014

Abstract. The workshop has focused on important developments within the last few years in the point of view and methods of real and harmonic Analysis as well as significant concurrent progress in the application of these to various other fields.

Mathematics Subject Classification (2010): 42B20.

Introduction by the Organisers

This workshop, which continued the triennial series at Oberwolfach on Real and Harmonic Analysis that started in 1986, has brought together experts and young scientists working in harmonic analysis and its applications (such as nonlinear dispersive and elliptic PDE, number theory, geometric measure theory) with the objective of furthering the important interactions between these fields.

Major areas and results represented at the workshop are:

• Fourier restriction theorems and Strichartz estimates.
• The study of sharp constant estimates for classical inequalities such as Hausdorff Young inequalities or restriction inequalities has lead to approaches to these inequalities quite different from those merely aiming at existence proofs for constants. These methods include group theoretic methods and special functions, variational methods, and very fine geometric arguments.
• Uniformity questions for oscillatory integrals play a role in various contexts in mathematics, the workshop has seen a discussion of progress on uniform lower bounds on Bergman kernels, a coordinate free approach to uniform
oscillatory estimates, and uniform bounds for Fourier restriction operators on polynomial curves. Uniformity in the dimension of maximal operator bounds is a related area of recent interest.

- Discrete analogues of results in harmonic analysis and connections to number theory.

- Multilinear singular integral theory in several dimensions studies operators formed by integrating a multidimensional singular kernel against a product of functions, each of these functions factoring through its own low dimensional projection of the integration domain. The behavior of the multilinear singular integral depends critically on the dimensionality and relative position of these projections, leading to a variety of interesting phenomena. These phenomena are only very partially understood, recent progress has been on various generalizations of commutator estimates and on very singular operators of entangled type.

- Analysis on spaces of Carnot-Caratheodory type, estimates on the Heisenberg group.

- A number of presentations of the workshop discussed various applications of harmonic analysis to PDE and further areas of mathematics. Applications included the magnitude of balls arising in category theory, calculated by solving a PDE, weighted integrability of polyharmonic functions, and a uniqueness theorem of Holmgren, and a Paley Wiener type theorem for Schrödinger evolutions, and inverse spectral theory for unbounded domains, and a discussion of minimal surfaces and sets.

- The interplay between martingale methods and harmonic analysis, for example to obtain sharp weighted estimates on singular integrals, including the recent progress on characterizing the two weight bounds for the Hilbert transform by testing conditions. The methods are also applicable in the study of questions in geometric measure theory, which require understanding of singular integral theory in very hostile environments such as spaces not of homogeneous type.

The meeting took place in a lively and active atmosphere, and greatly benefited from the ideal environment at Oberwolfach. It was attended by 53 participants. The program consisted of 28 lectures of 40 minutes. The organizers made an effort to include young mathematicians, and greatly appreciate the support through the Oberwolfach Leibniz Graduate Students Program, which allowed to invite several outstanding young scientists.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”.
## Workshop: Real Analysis, Harmonic Analysis and Applications

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