

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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## Geometrie

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ABSTRACT. The topics discussed at the meeting ranged from geometric evolution equations to minimal surfaces, Riemannian foliations and hyperbolic geometry. Because of a flexible schedule, the 53 participants had ample time for discussion.

*Mathematics Subject Classification (2010):* 53xx.

### Introduction by the Organisers

The format of the meeting consisted of 18 one hour talks and four half hour after-dinner talks. The after-dinner talks were given by PhD students and recent PhDs.

Six of the talks were related to geometric flows. Gerhard Huisken investigated the mean curvature flow with surgery in 3 dimensional manifolds. If one starts with a mean convex initial surface then there are only finitely many singularities and, in the case of long time existence, the solution converges to a stable minimal hypersurface of the ambient manifold. Carlo Sinestrari established various results on ancient solutions of the mean curvature flow, e.g. he gave several characterizations of the shrinking sphere solutions. Anton Petrunin suggested that each compact polyhedral space with nonnegative curvature might be the initial singular metric of a smooth orbifold Ricci flow with nonnegative curvature operator. He provided a proof in the 3-dimensional case. Tobias Marxen talked on the asymptotics ( $t \rightarrow \infty$ ) of the Ricci flow on a noncompact  $(n + 1)$ -dimensional manifold endowed with an isometric  $T^n$ -action. Peter Topping investigated the gradient flow of the Dirichlet energy of mappings from a surface  $S$  of genus  $\geq 2$  into a fixed Riemannian manifold  $M$ , letting both the map and the (hyperbolic) metric of the domain vary. For a nonpositively-curved target, he could establish long

time existence of the flow. Valentino Tosatti talked about the Kähler Ricci-flow and proved a decade-old conjecture on the set of singularities, namely it forms an analytic variety.

Kähler geometry was also the subject of two other talks. Hans-Joachim Hein gave a characterization of Stenzel's metric, a Ricci flat Kähler metric on the tangent bundle of the sphere. Ben Weinkove generalized Yau's solution of the Calabi conjecture to certain Hermitian metrics on closed manifolds.

Karl-Theodor Sturm gave a survey on synthetic definitions of lower Ricci curvature bounds on metric measure spaces, in terms of properties of the optimal mass transport and solutions of the heat equation. André Neves showed how min-max techniques can be used to settle Yau's question about infinitely many minimal hypersurfaces in 3-manifolds, in the case of positive Ricci curvature, along with higher-dimensional generalizations. Claude LeBrun established optimal estimates for the  $L^2$ -norm of the positive part of the Weyl curvature, for various classes of 4-manifolds. Esther Cabezas-Rivas showed that a Riemannian manifold with a lower sectional curvature bound and an upper diameter bound is finitely covered by a nilmanifold, provided that the  $L^1$ -norm of the curvature operator is sufficiently small.

There were three talks related to (singular) Riemannian foliations and isometric group actions. Marco Radeschi explained how Clifford representations can be used to find many nonhomogeneous singular Riemannian foliations of a round sphere. Alexander Lytchak proved rigidity statements for Riemannian foliations, ensuring that no exceptional fibers can occur, e.g. if the ambient space is a topological sphere and the leaf dimension is 7. Wolfgang Spindeler showed that fixed point homogeneous nonnegatively-curved manifolds admit a double disc bundle decomposition.

Ricardo Mendes explained why most known examples of homogeneous manifolds with positive sectional curvature also satisfy a certain stronger curvature condition, namely one can find metrics on these manifolds whose curvature operator can be modified by a four form in such a way that the modified curvature operator is nonnegative.

The geometry of hyperbolic space and higher rank symmetric spaces entered into three talks. Bernhard Leeb introduced the concept of a Morse action on a non-compact symmetric space, and used it to give a higher-rank substitute for convex cocompactness. Ursula Hamenstädt gave a simplified proof of the statement that any fundamental group of a hyperbolic 3-manifold contains a nontrivial surface subgroup, along with higher dimensional generalizations. Michelle Bucher gave integrality results for characteristic numbers of certain representations of lattices in hyperbolic isometry groups.

The remaining talks were given by Lange, Hensel and Gaifullin. Christian Lange answered the question of when the underlying topological space of an orbifold is in fact a manifold. Sebastian Hensel considered piecewise isometric self-maps of

an interval to itself and showed that the uniquely ergodic maps form a path connected subset. Alexander Gaifullin presented various methods to construct flexible polyhedra and showed that the enclosed volume stays constant under deformation.

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