

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Stochastic Analysis in Finance and Insurance

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ABSTRACT. This workshop brought together leading experts and a large number of younger researchers in stochastic analysis and mathematical finance from all over the world. During a highly intense week, participants exchanged many ideas during talks and discussions, and laid foundations for new collaborations and further developments in the field.

Mathematics Subject Classification (2010): 91GXX, 60XX.

Introduction by the Organisers

The workshop *Stochastic Analysis in Finance and Insurance*, organised by René Carmona (Princeton), Martin Schweizer (Zürich) and Nizar Touzi (Paris), was held May 4 – May 10, 2014. The meeting had a total of 50 participants from all over the world with a deliberately chosen mix of more experienced researchers and many younger participants.

During the five days, there were a total of 24 talks with many lively interactions and discussions. In addition, there were three blocks of short communications, as explained below.

The topics presented in the talks covered a very wide spectrum. Some of the major developments included a focus on optimal transport problems in connection with robust pricing and hedging, microstructure and other modelling issues, aspects of numerical computations in high-dimensional systems, and as always a number of foundational questions. To stimulate discussions and maximise interactions, talks were deliberately not organised into groups by major topics. A short overview of the talks given day by day looks as follows.

Freddy Delbaen in the first talk of the workshop presented some new structure results on monetary utility functions (or equivalently risk measures) with a view towards requirements imposed or discussed by regulators and practitioners. *Miklós Rásonyi* presented new results on hedging, arbitrage and optimality for portfolio choice in financial markets with superlinear frictions. *Jan Obłoj* discussed a robust approach to pricing and hedging and showed how trading restrictions can lead there to the emergence of financial bubbles. *Josef Teichmann* gave a new convergence result for the Emery topology on semimartingales and explained how this could be used to give a streamlined and structured proof of the fundamental theorem of asset pricing. *Chris Rogers* proposed a very simple Bayesian approach to inference and action in financial econometrics which deals in a simple and unified way with a number of otherwise not well addressed issues. Finally, *Olivier Guéant* combined ideas from optimal control and option pricing to account for effects arising from execution costs and market impact.

Jean Jacod started the second day with an overview of backward stochastic differential equations (BSDEs) driven by a multivariate point process, and showed how such equations can be solved in a pathwise manner by a kind of backward recursion. *Yuri Kabanov* presented a new result on tradable local martingale deflators which could be viewed as a weak formulation for a fundamental theorem of asset pricing in frictionless markets. *Sebastian Herrmann* proposed a simple and tractable model for optimal investment in a setting where the underlying asset price process has a bubble, described by a combination of a Black–Scholes type model with a single-jump local martingale. He showed how this problem could be solved fairly explicitly and gave rise to a number of interesting phenomena. *Nicole El Karoui* used stochastic progressive utilities to describe and analyse models for long-term decision making (e.g. for question about mortality or pension funds) in a stochastic environment.

Tuesday afternoon was devoted to the traditional excursion to Sankt Roman; this was moved forward by one day from the usual Wednesday afternoon schedule in view of the weather forecast for the second half of the week (and this decision turned out to have been very wise).

On Wednesday, *Mete Soner* presented a continuous-time duality for robust hedging in models where price processes are assumed to be RCLL (or càdlàg). *Christoph Czichowsky* explained how optional strong supermartingales arise in the treatment via duality techniques of the problem of optimal portfolio choice under transaction costs. In addition, there were a number of short communications in a format which was introduced (and judged to be very successful) in an earlier meeting. Each presenter had 10 minutes to explain his result, which were then followed by 5 minutes of questions and discussion. This idea of explaining in a nutshell some current problems or results again met with great success; the list of speakers for giving a short presentation very quickly grew to a total of 15 names, and the corresponding talks were scheduled on Wednesday morning, Thursday morning and Thursday afternoon. Wednesday afternoon continued with *Terry Lyons* who showed how one could use sophisticated mathematical tools to extract

in a very systematic way information from sequential data, without any a priori knowledge of the data or even the information contained in it. Finally, *François Delarue* gave a derivation of the master equation arising in mean field games and sketched a way to prove the existence of a classical solution to this forward-backward system of PDEs.

Thursday started with *David Hobson* who presented a number of results on fake diffusions, i.e. martingales whose univariate marginals (which in financial terms are determined by the prices of plain vanilla options) match a given family of probability measures with certain properties (again dictated by financial requirements). *Tom Hurd* presented the Gai–Kapadia model for systemic risk and gave some formal computations for obtaining the default probability distribution after a number of default cascade steps. A second block of short communications followed, leading again to intense discussions that continued into the afternoon and in the evenings. *Dan Lacker* then gave some new results on mean field games with a common noise term, which appear in (approximate) equilibria of symmetric stochastic differential games. *Umut Çetin* presented some recent developments in the microstructure models of Kyle and Glosten–Milgrom. The day was closed by a third block of short communications.

On the last day, *Xiaolu Tan* discussed martingale optimal transport with constraints on the one-dimensional marginals and explained some new ideas on how to connect such problems to Skorohod embeddings. *Arturo Kohatsu-Higa* gave a probabilistic representation of the parametrix method and explained how this could be used for numerical computations e.g. of prices for exotic options. *Shigeo Kusuoka* gave an overview of Monte Carlo methods for pricing Bermuda derivatives, highlighting in particular advantages and shortcomings of some alternative but competing approaches. *Monique Jeanblanc* presented a number of examples to illustrate some very subtle issues arising in the study of arbitrage theory in connection with an enlargement of filtration, as for example needed in the context of credit risk. *Dylan Possamai* studied the properties of the solutions of backward stochastic differential equations (BSDEs) with a view towards obtaining in particular the existence of a density for both the state as well as the integrand processes. Finally, *Mathieu Rosenbaum* gave a number of limit theorems for nearly unstable Hawkes processes that appear in the context of microstructure modelling in financial markets.

Like in the workshop three years before, there were an enormous number of discussions, interactions and exchanges. Everyone felt privileged to be able to spend a highly productive and creative week at the unique place that has been created in Oberwolfach, and to profit from the excellent infrastructure, support and scientific environment. In particular, the younger participants and the first-time visitors to Oberwolfach unanimously said that the actual experience of the workshop and the overall scientific atmosphere still exceeded their already high anticipations.

As organisers and on behalf of all participants, we want to express, like in the last workshop three years earlier, our gratitude to the Mathematisches Forschungsinstitut Oberwolfach for giving us the opportunity of having this very successful workshop. We hope that we shall be able to come back at some time in the future.

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René Carmona
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