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Nonlinear Evolution Equations: Analysis and Numerics

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Abstract. The workshop was devoted to the analytical and numerical investigation of nonlinear evolution equations. The main aim was to stimulate a closer interaction between experts in analytical and numerical methods for areas such as wave and Schrödinger equations or the Navier–Stokes equations and fluid dynamics.

Mathematics Subject Classification (2010): 35xx, 65xx.

Introduction by the Organisers

The qualitative theory of nonlinear evolution equations is an important tool for studying the dynamical behavior of systems in science and technology. A thorough understanding of the complex behavior of such systems requires a detailed analytical and numerical investigation of the underlying partial differential equations. Here one is interested in regularity and asymptotic properties of solutions as well as in efficient numerical approximations of the solutions which preserve their qualitative properties on a large time scale. Currently, driven by the challenging mathematical difficulties and supported by the influx of techniques from many branches of analysis and numerics, innovative and sophisticated methods have been developed throughout this area of mathematics.

This workshop has focused on recent developments in the qualitative theory of nonlinear evolution equations, both analytical and numerical, between which there has been an increasing interaction in recent years. One of the main goals of this conference was to bring together international experts with different backgrounds in analysis and numerics to stimulate the transfer of ideas, results, and techniques among them. To this aim, the speakers reported on new trends in the fundamental classes of evolution equations, such as

- wave equations,
- Schrödinger equations,
- Maxwell's equations,
- Navier–Stokes equations and fluid dynamics,
- reaction-advection-diffusion equations,
- stochastic evolution equations.

The numerical study of evolution equations relies on efficient methods for time integration which allow for convergence results not depending on spatial discretization parameters (such as mesh size or number of basis functions). For the rigorous derivation of finite time error bounds and the study of geometric properties, one has to develop a framework for the discretization that captures the essential properties of the analytic problem. This makes a thorough understanding of the partial differential equation itself indispensable. Therefore, the various approaches in this field are closely linked with the analytic theory of evolution equations: splitting and approximation schemes can be formulated and treated within semigroup theory, exponential integrators are directly connected to functional calculi, and spectral methods heavily employ Fourier analysis. The proof of rigorous error bounds typically relies on regularity theory. This close relationship also plays a crucial role for geometric integrators which are designed to capture the qualitative properties of solutions (such as long term behavior or positivity).

In this spirit, the talks treated the long time behavior of fluid interaction models, geometric evolution equations and wave maps. The role of Strichartz estimates in dispersive problems was explored, and maximal regularity for deterministic and stochastic parabolic evolution equations was investigated. Space-time discretizations as well as semidiscretizations of evolution equations were discussed. Techniques from asymptotic analysis were exploited to construct and analyze multiscale problems. (Non-)Linear semigroup theory and Sobolev space theory were shown to be an enable the analysis of splitting methods and exponential integrators for Korteweg–de Vries and nonlinear Schrödinger equations. Convolution quadrature was successfully used for the numerical discretization of problems on unbounded domains. The presentations and the fruitful discussions demonstrated the far-reaching potential of further exchanges between analytical and numerical approaches to nonlinear evolution equations.

Workshop: Nonlinear Evolution Equations: Analysis and Numerics

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