Abstract. The earliest work in topology was often based on explicit combinatorial models – usually triangulations – for the spaces being studied. Although algebraic methods in topology gradually replaced combinatorial ones in the mid-1900s, the emergence of computers later revitalized the study of triangulations. By now there are several distinct mathematical communities actively doing work on different aspects of triangulations. The goal of this workshop was to bring the researchers from these various communities together to stimulate interaction and to benefit from the exchange of ideas and methods.

Mathematics Subject Classification (2000): 52B, 57M (Primary); 05A, 05E, 11H, 14M, 14T, 52C, 53A, 57N, 57Q, 68U (Secondary).

Introduction by the Organisers

The workshop Triangulations, organised by William H. Jaco (Stillwater), Frank H. Lutz (Berlin), Francisco Santos (Santander) and John M. Sullivan (Berlin) was held April 29th – May 5th, 2012. The meeting was well attended, with 53 participants from 14 countries (including Argentina, Australia and Israel). Besides the 27 lectures, the program included evening sessions on mathematical software and on open problems. The workshop successfully brought the different communities of mathematicians interested in triangulations together, resulting in several new collaborative projects between mathematicians who had never met before.

Triangulations have become increasingly important in both discrete geometry and manifold topology, but this work has proceeded independently without much interaction between the communities. Even the word “triangulation” can be a
source of confusion: for 3-manifolds the most general pseudo-simplicial triangulations (typically with a single vertex) are preferred, but discrete geometers mostly restrict to simplicial complexes, while in polytope theory and computational geometry these must be linearly embedded. Thus for instance while Pachner moves (bistellar flips) are useful to pass from one triangulation to another (as highlighted in the talks by de Loera and Burton), the moves available and their exact properties depend on the class of triangulations considered.

The combinatorial approach through 0-efficient triangulations and normal surfaces has introduced methods from geometric analysis into the combinatorial study of 3-manifolds. The resulting enumeration and decision problems are important and challenging examples in the study of computational complexity (as shown in the talks by Schleimer and Hass).

Complexity measures for 3-manifolds are well established in the pseudo-simplicial world (see the talks by Matveev, Martelli, Casali and Tillmann), but there is also a possible analogue in the combinatorial triangulation world (as proposed in the talk by Swartz).

Within discrete geometry, tools of algebraic topology and even algebraic geometry are often essential for answering fundamental questions. For instance, Stanley’s proof of the $g$-theorem (characterizing the $f$-vectors of polytopes) is based on the cohomology of toric varieties. Recent progress along these lines was reported by Joswig, Nevo and Swartz.

One example of a technique used in both communities to relate geometry and combinatorics is putting CAT(0) metrics on triangulations, as mentioned in the talks by Rubinstein, Benedetti and Adiprasito.

The workshop schedule left plenty of free time for informal interactions, and many fruitful discussions developed between mathematicians who had just met for the first time. For instance, knowledge about simplicial decompositions of the dodecahedron led to new insights on minimal triangulations of the Seifert–Weber dodecahedral space. As another example, less than two months after the workshop, Hähnle, Klee and Pilaud posted a preprint (arXiv:1206.6143) on weak decomposability based on work started at the Oberwolfach workshop.
Workshop: Triangulations

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