Abstract. The workshop “Mathematical Logic: Proof Theory, Constructive Mathematics” was centered around proof-theoretic aspects of current mathematics, constructive mathematics and logical aspects of computational complexity.

Mathematics Subject Classification (2000): 03Fxx.

Introduction by the Organisers

The workshop Mathematical Logic: Proof Theory, Constructive Mathematics was held November 6-12, 2011 and included three tutorials:

1. Sergei N. Artemov: Provability vs. computational semantics for intuitionistic logic (2 times 1 hour),
2. Jan Krajíček: Search for hard tautologies (3 times 1 hour),
3. Angus MacIntyre: Issues around proving Fermat’s Last Theorem (FLT) in Peano Arithmetic (3 times 1 hour).

Artemov’s tutorial gave an introduction to the provability semantics (based on explicit proof polynomials) for intuitionistic logic as developed for propositional logic by the author since 1995 and its very recent 2011 extension to a first-order logic of proofs by himself and T. Yavorskaya. Krajíček’s tutorial gave a survey on recent developments in the area of proof complexity and bounded arithmetic. MacIntyre presented some of the key ingredients of Wiles’ proof of FLT and outlined how the necessary mathematics could in principle be formalized in a suitable conservative extension of Peano Arithmetic PA.
In addition to these tutorials, 24 short talks of 25 minutes were given aiming:

*To promote* the interaction of proof theory with core areas of mathematics via the use of proof theoretic techniques to unwind ineffective proofs in mathematics. Two talks (L. Leuștean, P. Safarik) reported on recent extractions of explicit rates of metastability (in the sense of T. Tao) from proofs in nonlinear ergodic theory, while J. Gaspar gave an unwinding of a proof in metric fixed point theory and reported on proof-theoretic results concerning different finitizations (again in the sense of Tao) of the infinite pigeonhole principle. Applications of the method of cut-elimination to proofs in theories with axioms having a suitable logical form were the subject of another talk (S. Negri). A. Kreuzer talked about a proof-theoretic analysis of important principles in Ramsey theory and their connection to analytic principles. Other talks focussed on the more theoretical side of proof interpretations such as a novel functional interpretation for nonstandard analysis (B. van den Berg), refined negative interpretations (H. Ishihara), connections between Spector’s bar recursion and methods to compute Nash equilibria based on products of selection functions (P. Oliva) and recent developments on the ε-substitution method (G. Mints). P. Schuster gave a constructive reformulation of certain type of ineffective proofs in algebra while D.S. Bridges talked about different constructive formulations of the Riemann series theorem. A talk by H. Schwichtenberg was concerned with a novel inductive/coinductive treatment of continuous functions. Real numbers as an abstract data type for the extraction of programs from proofs was presented by A. Setzer.

*To further develop* foundational aspects of proof theory and constructive mathematics. Two talks (G.E. Leigh, T. Strahm) investigated proof-theoretic properties of theories of truth. Other talks dealt with new conservation results for systems of constructive set theory (L. Gordeev) and recent developments in Voevodsky’s program of ‘univalent foundations’ which was the subject of another Oberwolfach workshop in October (P. Aczel). Two further talks discussed approaches to ordinal notations based on reflection principles (L. Beklemishev) and ‘patterns of resemblance’ due to T.J. Carlson (G. Wilken), respectively. A talk by G. Jäger investigated the proof-theoretic strength of operational systems of set theory (formulated in the framework of Feferman’s explicit mathematics). A talk by A. Visser was concerned with the provability logic of arithmetics.

*To explore* further the connections between logic and computational complexity: this concerns both proof-theoretic results of systems of bounded arithmetic (A. Beckmann) as well as an understandind of what makes certain formulas hard for current SAT-solvers while even very large formulas stemming from concrete applications often can be decided rather efficiently by these tools. (J. Nordström). An axiomatic approach to the issue of the intrinsic complexity for general classes of algorithmic problems in arithmetic and algebra in terms of absolute lower bounds was developed in a talk by Y. Moschovakis based on a so-called homomorphism method.
# Workshop: Mathematical Logic: Proof Theory, Constructive Mathematics

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