

MATHEMATISCHES FORSCHUNGSIINSTITUT OBERWOLFACH

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**Algebraische Zahlentheorie**

Organised by

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**ABSTRACT.** The workshop brought together researchers from Europe, Japan and the US, who reported on various recent developments in algebraic number theory and related fields. Dominant topics were Shimura varieties, automorphic forms and Iwasawa theory.

*Mathematics Subject Classification (2000):* 11R, 11S.

**Introduction by the Organisers**

The workshop *Algebraische Zahlentheorie*, organised by Benjamin Howard (Chestnut Hill), Guido Kings (Regensburg), Ramdorai Sujatha (Bombay) and Otmar Venjakob (Heidelberg) was well attended with 55 participants from Europe, Japan and the US. In total we had 19 talks on various topics such as  $p$ -adic Hodge theory, Galois representations and  $p$ -adic representation theory, automorphic forms, Shimura varieties, Iwasawa theory etc.

This time the Algebraic Number Theory Workshop was completely dominated by tremendous results and talks of young mathematicians. To start with, Peter Scholze described the theory of perfectoid spaces, a general framework for questions of changing between equal- and mixed-characteristic local fields, which leads to an improvement on Faltings's almost purity theorem as well as the proof of a new important special case of the weight-monodromy conjecture  $l$ -adic cohomology. His work is similarly based on Huber's adic spaces as Eugen Hellmann's new approach to “arithmetic families of filtered  $\phi$ -modules and crystalline representations generalising vastly Kisin's weakly admissible filtered  $\phi$ -modules. Moritz

Kertz contributed a strong result (jointly with Spencer Bloch, Hélène Esnault) on the formal deformation part of  $p$ -adic variational Hodge conjecture.

Another group of talks concerned Iwasawa theory. Jonathan Pottharst and Antonio Lei (joint work with Sarah Zerbes and David Loeffler) each proposed new approaches for Selmer groups and related main conjectures in the non-ordinary case: while Pottharst considers families of Galois representations and uses  $(\phi, \Gamma)$ -modules over the Robba-ring, Lei considered Galois representations attached to weight  $k$  normalised eigen-newforms using Wach-modules. Takako Fukaya reports on a joint work with Kazuya Kato concerning the proof (in certain cases) of a conjecture by Sharifi, which he had presented in Oberwolfach during an earlier Algebraic Number Theory workshop and which relates a cup-product pairing in Galois cohomology with  $L$ -values of certain cusp forms. Ming-Lun Hsieh reported on strong results towards the Iwasawa Main Conjecture for CM fields and their descent implications towards the ( $p$ -adic) Birch&Swinnerton-Dyer Conjecture. Thanasis Bouganis used similar automorphic techniques in order to show (the first step) of those congruences among certain abelian  $p$ -adic  $L$ -functions attached to unitary groups which show up in the work of Kakde and Ritter & Weiss to indicate the existence of non-abelian  $p$ -adic  $L$ -functions. Finally Cornelius Greither describes joint work with Cristian Popescu on Fitting ideals associated with 1-motives over global fields and related Equivariant Main Conjectures, in particular they obtain explicit constructions of Tate sequences.

With respect to another dominating topic, viz Galois representations and  $p$ -adic representation theory (towards  $p$ -adic local Langlands), we had first of all Gaëtan Cheneviers report on joint work with Jean Lannes concerning the classification of certain Galois representations of dimension 16 and 24 related to the set of isometry classes of even unimodular lattices in the standard euclidean space of the same dimensions. While Jan Kohlhaase reported on joint work with Benjamin Schraen on “Homological vanishing theorems for locally analytic representations”, Florian Herzig reported on “Weights in a Serre-type conjecture for  $U(3)$ ”, joint work with Matthew Emerton and Toby Gee on generalisations of Serre’s conjectures if the reductive group is an outer form of  $GL_3$ .

Concerning  $p$ -adic Hodge theory Takeshi Tsuji discussed extensions of the functor  $D_{\text{crys}}$  to the category of  $p$ -adic perverse sheaves with singularities along a simple normal crossing divisor. Jan Brunier described an interpretation of the coefficients of the  $q$ -expansion of certain weight 1/2 harmonic weak Maass forms in terms of Heegner divisors.

Also there were a couple of talks concerning Shimura varieties and period spaces beginning with the talk of Michael Rapoport, one of the few senior speakers, on the Arithmetic Fundamental Lemma of (and jointly with) Wei Zhang. Eva Viehmann presented the proof of a conjecture by Harris on the cohomology of Rapoport-Zink spaces being parabolically induced from that of a smaller moduli space while Fritz Hörmann and Keerthi Shyam Madapusi Sampath talked about heights of special cycles on and certain compactifications of Shimura varieties, respectively.

Bianca Viray discussed transcendental elements in Brauer groups of elliptic surfaces.

Finally it is perhaps worth mentioning that the organisers were quite relieved about having postponed the traditional hiking tour (this time to St. Roman) from Wednesday to Thursday afternoon, thereby avoiding heavy thunderstorms.

