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Stochastic Analysis

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Abstract. The meeting took place on May 30 - June 3, 2011, with over 55 people in attendance. Each day had 6 to 7 talks of varying length (some talks were 30 minutes long), except for Thursday: the traditional hike was moved to Thursday due to the weather (and weather on thursday was indeed fine).

The talks reviewed directions in which progress in the general field of stochastic analysis occurred since the last meeting of this theme in Oberwolfach three years ago. Several themes were covered in some depth, in addition to a broad overview of recent developments. Among these themes a prominent role was played by random matrices, random surfaces/planar maps and their scaling limits, the KPZ universality class, and the interplay between SLE (Schramm-Loewner equation) and the GFF (Gaussian free field).

Mathematics Subject Classification (2000): 60Gxx, 60Jxx,82Bxx.

Introduction by the Organisers

The workshop opened in a bang with a description, by J.-F. Le Gall and by G. Miermont, of the recent proofs of the universal convergence (in the Gromov-Hausdorff metric) of random planar maps (q-angulations, with $q = 3, 2k$ with k integer) toward the Brownian map, an object identified earlier by J.-F. Le Gall. The importance of confluence of geodesics was stressed in both talks. These talks were completed by a talk of M. Bousquet-Mélou on combinatorial aspects of the Potts model on planar maps, and later in the week, by B. Eynard who derived general (universal) equations for the enumeration of maps and other objects, and discussed the link with random matrices. Later in the week, N. Curien presented natural examples (joint work with Le Gall and with Werner) of triangulations of the unit disc (splittings of the unit disc into triangles that have their three

corners on the boundary of the disc) of a different type than the "uniform" random triangulation that had been studied by Aldous and that plays an important role in the understanding of planar maps.

Several talks discussed random surfaces and models from the SLE perspective. S. Sheffield introduced the "quantum zipper", that allows to sew together two random surfaces along an SLE curve – here the random surfaces are defined in a generalized sense via the Gaussian free field, that is a rather central object in the study of continuous two-dimensional random geometries; Sheffield also discussed the link with work in progress with J. Miller and with B. Duplantier. Later in the week, J. Miller reported on his work with Sheffield concerning the identification of the geometry of "altimeter-compass lines" and "light cones" within the geometry defined via the Gaussian free field. J. Dubédat addressed questions related to dimers. In the direction pioneered by Kenyon on dimer configurations in planar graphs, he explained how when one controls analytically the quantities involved, one can get powerful results by estimating the behavior of suitably perturbed Laplacians and their determinant, in the scaling limit when the mesh-size of the lattice vanishes. This is one of the cases where the scaling limit of discrete models on discrete graphs can be connected to continuous limitting structures such as SLE curves and the Gaussian free field. V. Vargas recalled results by Jean-Pierre Kahane on mulitplicative cascades and constructions of limiting measures such as the one appearing in Sheffield's lecture (the "exponential of the Gaussian free field") and that is conjecturally related to the Brownian map, and his recent work with Allez and Rhodes, that generalizes the construction of characterization of these measures for continuous cascades.

C. Garban described scaling limits for magnetization in the Ising model at criticality (where non-trivial scaling exponents appear), and G. Pete used again SLE methods to study near critical dynamics for the planar FK Ising model.

More classical topics related to percolation and Ising models were also present: G. Grimmett presented his recent work with I. Manolescu that enables to bound crossing probabilities of boxes for a wide class of critical planar percolation models, and A. Holroyd described his joint work with Grimmett on aspects of the geometry of supercritical percolation clusters (can one embed in a Lipschitz way a two-dimensional plane into a three-dimensional cluster etc.). In a different direction, H. Lacoin described a derivation of an upper bound on relaxation times for the zero temperature stochastic Ising dynamics. C. Bordenave described his joint work with Lelarge and Salez on the understanding of random configurations of dimers on a discrete graph, in the scaling limit (questions like "what is the asymptotic density of holes in such configurations"?).

Another cluster of talks discussed recent progress around scaling limits for models inspired by first passage percolation and the KPZ universality class. T. Seppalainen described an explicitely solvable model of a directed polymer with gamma weights, and his talk was continued by I. Corwin who reported on a follow up joint work with O'Connell, Seppalainen and Zygouras that uses a geometric RSK correspondence, a criterion of Rogers and Pitman, and Whittaker functions, to give

a Fredholm determinant representation for endpoint fluctuations of a family of directed polymers, including the gamma-weighted one. The KPZ theme was taken up by Sasamoto, who described his results on convergence to the KPZ equation with appropriate initial conditions, and by J. Quastell, who gave an overview of his results (joint with Corwin, Remenik and Moreno) on fluctuations of extrema of the Airy_2 process around a parabolic barrier. He discussed a model of continuous Brownian polymer, studied by him, Alberts and Khanin, and its relation with the KPZ equation. Later in the week, H. Widom discussed his fundamental result with C. Tracy concerning the asymmetric exclusion process (ASEP), explaining an earlier gap in the proof and the way it is fixed, allowing for multi-type ASEP. Back on the first passage percolation theme, S. Chatterjee discussed his recent geometric proof of a universal relation between different scaling exponents.

A third cluster of talks was centered around random matrices and random Schroedinger operators. M. Aizenman and S. Warzel described their recent results on the boundary of the delocalization region for the random Schroedinger operator on the regular tree; this work revises the conjectured picture and provides a rigorous description of the boundary at weak disorder. Recursions of Green functions play a fundamental role in the proof. Another aspect of the spectrum of RSE was discussed by B. Virag, who reported on results with Kritchevski and Valko concerning convergence to a Brownian caroussel process for a 1-D RSE problem with scaled down potential, and to GOE statistics for a particular scaling of the RSE on a strip. A. Knowles described recent work with Erdos, Yau and Yin on universality results for the eigenvalues of the adjacency matrix of random Erdos-Renyi graphs, in the regime where the row sum goes to infinity. He introduced the steps, developed earlier by Erdos, Schlein, Yau and Yin, to prove universality for random matrices by deriving a local semi-circle law, (modified) Dyson flow and a matching lemma. This was followed up by H.-T. Yau, who gave more details on the Dyson flow and explained how that step can be bypassed in universal beta-ensembles by proving a version of local equilibrium for Gibbs measures.

Other talks given during the week covered other stochastic analysis themes. E. Bolthausen described his his joint work with F. Rubin on the asymmetric weakly self-avoiding walk in high dimension, and the use of appropriate recursions and induction to prove a CLT. H. Duminil-Copin gave an essentially complete proof of a recent work in progress with Benjamini, Kozma and Yadin concerning the control of coupling (and hence, Harmonic functions) by entropic methods for random walks on a variety of graphs. J.-D. Deuschel described his work with Berger on the invariance principle (quenched) for certain non-elliptic environments. A. Hammond talked about his joint work with Fribergh on biased random walk in random environment (such as supercritical percolation clusters) that allows to describe and understand the transition between a ballistic regime (when the drift is not too large) to slow regime (when the drift is too large, the walk is slowed down by traps). A. Bovier discussed the limiting law of the particles in a branching Brownian motion viewed from the leading edge, obtained with Arguin and Kistler, and explained the spin-glass motivation behind this work. T. Kumagai discussed

an approach, based on the notion of spectral Gromov-Hausdorff distance, that allows to prove convergence of (L^p) mixing times on a family of graphs to the mixing time of a diffusion on a limiting object.