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Finite-dimensional Approximations of Discrete Groups

Organised by
Goulnara Arzhantseva, Wien
Andreas Thom, Leipzig
Alain Valette, Neuchâtel

May 15th – May 21st, 2011

Abstract. The main objective of this workshop was to bring together experts from various fields, which are all interested in finite and finite-dimensional approximations of infinite algebraic and analytic objects, such as groups, algebras, dynamical systems, group actions, or even von Neumann algebras.

Mathematics Subject Classification (2000): 05C25, 20F20, 20F32, 22D40 (18B40, 20E05, 22E15, 22E40).

Introduction by the Organisers

The workshop *Finite-dimensional Approximations of Discrete Groups*, organized by Goulnara Arzhantseva (Wien), Andreas Thom (Leipzig) and Alain Valette (Neuchâtel) was held May 15th – May 21st, 2011. This meeting was well attended with 25 participants. Many of the participants are young researchers which just established themselves. This gave the workshop a very active atmosphere. Open discussions during talks were vivid and productive.

Over the last decade, finite approximation of groups has become an important ingredient in the understanding of various longstanding conjectures. A group is called *sofic* if its group law can be approximated on finite sets by finite permutations up to any given ε, where errors in the symmetric group are measured in terms of the usual Hamming distance. A series of breakthrough applications of this notion was started with Misha Gromov's solution of Gottschalk's Surjunctivity Conjecture for sofic groups. Later, Gabor Elek and Endre Szabo proved the Kaplansky Finiteness Conjecture for sofic groups. More recently, it was observed that Kervaire's Conjecture holds for hyperlinear groups; a class of groups which

admits a modeling of the group law by unitary matrices. The notion of soficity has triggered groundbreaking work by Lewis Bowen, which resulted in a definition of entropy for actions of sofic groups, extending the existing definitions for amenable groups. The class of hyperlinear groups is well-studied by the operator-algebraic community, since it naturally arises from the definition of free entropy dimension given by Dan Voiculescu, which is based on matricial microstates.

The aim of the workshop was to bring together experts from various fields which share interest in finite and finite-dimensional approximation of infinite algebraic and analytic objects, such as groups, algebras, dynamical systems, group actions, and von Neumann algebras.

Since the audience was mixed, we started the workshop with a series of survey talks. Andreas Thom, Miklos Abert, Lewis Bowen and Narutaka Ozawa gave lectures about various aspects of finite and finite-dimensional approximation. The survey talks were followed by research talks on the remaining days, some of which will be mentioned in this section. Łukasz Grabowski presented in his talk the recent solution to the Atiyah Problem in the torsion case. Building on work of Tim Austin, and working independently of the team Pichot-Schick-Zuk, he could show that *every* non-negative real number can be the kernel-dimension of an element in the integral group ring of a countable group. Adam Timar presented the first example of a sequence of Cayley graphs which is convergent in the sense of Benjamini-Schramm, which however admits no coloring, so that the resulting sequence of Cayley diagrams converges as a sequence of Cayley diagrams. Miklos Abert (joint work with Glasner-Virag) presented a variant of Kesten's Theorem for discrete measured groupoids, which among other things imlies that every sequence of Ramanujan graphs has a sub-linear number of k-cycles. Ken Dykema (joint work with Kerr-Pichot) presented the recent invention of the sofic-dimension of a group or more generally a discrete measured groupoid. Wolfgang Lück presented new results on torsion-growth and homology-growth in the unexplored territory of homology in finite characteristic. Simon Thomas lectured about structure results about the ultraproduct of alternating groups, which shed light on the universal sofic group. Mark Sapir gave an introduction to the Higman embedding theorem and proved a new variant of it, which implies that expander graphs can be coarsely embedded into fundamental groups of manifolds. Erik Guentner (joint with Arzhantseva-Spakula) reported about coarse embeddings and related topics. Outlines of these and various other interesting talks appear with extended abstracts in this report.

The workshop included various evening sessions with additional talks and an extensive problem session on Thursday afternoon. After positive feedback from all sides, we (the organizers) came to the conclusion that the workshop achieved its ambitious goal of establishing contact across various areas of research.