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Geometric Methods of Complex Analysis

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Abstract. The purpose of this workshop was to discuss recent results in Several Complex Variables, Complex Geometry and Symplectic/Contact Geometry with a special focus on the interplay and exchange of ideas among these areas, which proved to be very fruitful in the last years. The main topics of the workshop included Symplectic and Contact Geometry, Almost Complex Geometry, Pluripotential Theory and the Monge-Ampère equation, Complex Dynamics, Geometric Questions of Complex Analysis (including Theory of Foliations) and Applications, the $\bar{\partial}$-equation and Geometry.

Mathematics Subject Classification (2000): 32xx, 53xx, 14xx, 37Fxx.

Introduction by the Organisers

The workshop *Geometric Methods of Complex Analysis* attracted 53 researchers from 16 countries. Both, leading experts in the field and young researchers (including five Ph. D. students) were well represented in the meeting and gave talks. Rather wide spectrum of topics related to Complex Analysis (and this was one of the aims of the workshop) was covered by the talks and unformal discussions. All 24 lectures presented on the meeting can be conditionally divided into the following groups.

Symplectic and Contact Geometry was represented by talks of H. Geiges and V. Shevchishin. Geiges explained Eliashberg's idea for proving Cerf's theorem which is based on the method of filling with holomorphic discs. Shevchishin gave a description of the diffeotopy group of a rational or ruled complex surface.

Almost Complex Geometry was represented by the talks of B. Saleur and A. Gournay (both being young researchers). Saleur described a generalization of the classical Borel's and Bloch's theorems to the case of a smooth almost complex structure on $\mathbb{P}^2$ which is tamed by the Fubini-Study form. Gournay presented a generalization of the Runge approximation theorem for the case of (pseudo-) holomorphic maps from a compact Riemann surface to a compact (almost-) complex manifold.

Pluripotential Theory and the Monge-Ampère equation is an important topic which was well represented on the meeting. Four lectures in this area were given by V. Guedj, S. Boucksom, L. Lempert and D. Coman. Guedj presented a solution of the analogue of the Calabi conjecture in a big cohomology class inspired by viscosity techniques. Boucksom explained a variational approach to complex Monge-Ampère equations which gives characterization of Kähler-Einstein metrics and has applications to the Kähler-Ricci flow. Lempert presented a result on nonexistence (in general) of geodesics connecting two given points in the space of Kähler metrics. This solves a long standing open problem connected to extremal metrics on Kähler manifolds. Coman described a result on extension of plurisubharmonic functions from analytic subvarieties with sharp growth control.

Complex Dynamics was represented by the talks of N. Sibony, H. Peters and E. Bedford. Sibony presented results on finiteness of entropy of a meromorphic map of a compact Kähler manifold and of foliations by Riemann surfaces. In the first case he has also provided a bound for entropy by the maximum of the logarithm of the dynamical degrees. Peters explained the role of limit varieties for a Fatou component for selfmaps in two complex variables. For holomorphic endomorphisms he gave a classification of the Fatou components under the assumption of uniqueness. Bedford discussed periodicities and positivity of entropy for linear fractional recurrences in 3-space.

Geometric Questions of Complex Analysis including Theory of Foliations and Applications were represented by the talks of S. Ivashkovich, H. Samuelsson, F. Kutzschebauch, D. Popovici, B. Jöricke, J. Globevnik, G. Bharali, G. Henkin, E. Rousseau and S. Orevkov. Ivashkovich presented (for any given integer $d \geq 1$) an example of a rational self-map $f : \mathbb{P}^2 \to \mathbb{P}^2$ of degree d without holomorphic fixed points. He also described different topologies on the space of meromorphic maps. Samuelsson described a generalization of classical theorems by Čirka and Axler-Shields to the multidimensional case. Kutzschebauch gave a complete positive solution of Gromov's Vaserstein problem. Namely, he proved the existence of holomorphic factorization of null-homotopic holomorphic mappings from a reduced Stein space into $SL_n(\mathbb{C})$ in a product of upper and lower diagonal matrices. Popovici presented the new concept of "strongly Gauduchon manifold" and explained how using this concept one can prove a long-standing conjecture: if all the fibres, except of one, of a holomorphic family of compact complex manifolds are projective, then the remaining fibre is Moishezon. Jöricke gave a sharp lower bound of the 4-ball genus of an arbitrary analytic knot L contained in a small tubular neighborhood of a given smoothly analytic knot K in terms of the 4-ball genus of K and the "Umlaufszahl" of L with respect to K. Globevnik characterized pairs of points a, b in $\mathbb{C}^2$ having the property that if a function $f \in \mathcal{C}^\infty(b\mathbb{B})$, where $\mathbb{B}$ is the open unit ball in $\mathbb{C}^2$, extends holomorphically inside $\mathbb{B}$ along each complex line passing either through a or through b, then f extends holomorphically to the whole of $\mathbb{B}$. Bharali explained how to achieve pseudoconvex bumping near a weakly pseudoconvex boundary point of some finite-type pseudoconvex domains in $\mathbb{C}^3$. Henkin presented recent results in the theory of complex Radon transforms and their applications. Rousseau described his work on holomorphic mappings $f : \mathbb{C}^p \to X$ of generic maximal rank into a projective manifold of dimension n, such that the image of f is tangent to a foliation $\mathcal{F}$ on X. He also discussed a generalized Green-Griffiths-Lang conjecture and presented several results of algebraic degeneracy in the strong sense. Orevkov explained how using the projective duality of plane projective complex curves one can give a complete solution to the problem of classification of systems of orthogonal polynomials in two variables.

The $\bar{\partial}$-equation and Geometry were represented by the talks of T. Ohsawa, J. Ruppenthal and M. Andersson. Ohsawa presented his results related to pseudoconvexity, the variation of the Bergman kernels and Levi flat manifolds which were based on refined L^2-theorems. Ruppenthal explained how L^2-Dolbeault cohomology groups $H^{0,q}_{(2)}(X - \mathrm{Sing}X)$ can be described by the cohomology of the sheaf of germs of meromorphic functions with poles according to a certain effective divisor on a resolution $\pi : N \to X$ of a singular space. Andersson discussed global division problems on algebraic varieties and presented generalizations to singular varieties of various results previously known for smooth varieties. He also gave an analytic proof of the Briançon-Skoda-Huneke theorem and combining the residue theory with integral formulas he obtained semiglobal Koppelman formulas for $\bar{\partial}$ on analytic spaces.