Geometric Quantization in the Non-compact Setting

Organised by
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February 13th – February 19th, 2011

ABSTRACT. The purpose of the workshop was to bring together mathematicians interested in "quantization of manifolds" in a broad sense: given classical data, such as a Lie group G acting on a symplectic manifold M, construct a quantum version, that is a representation of G in a vector space $Q(M)$ reflecting the classical properties of M.

Introduction by the Organisers

The workshop, *Geometric Quantization in the Non-compact Setting*, organized by Lisa Jeffrey (Toronto), Xiaonan Ma (Paris) and Michèle Vergne (Paris) was held February 13th - February 19th, 2011.

The meeting was attended by 48 participants, representing researchers from many European countries, and Australia, Canada, China, Japan, USA. Unfortunately, Lisa Jeffrey could not be present because a minor injury before the meeting made it impossible for her to travel.

The meeting was devoted to the following theme (and adjacent themes). Let G be a Lie group acting on a manifold M. Assume that G preserves some data (D), such as a symplectic structure, or a differential operator on M, or a fibration, ..., then the aim of Geometric Quantization is to associate to these data a representation of G in a vector space $Q(M, D)$, and to analyze the relations of the quantum space $Q(M, D)$ with the classical data (M, D). There are diverse constructions of the quantum space $Q(M, D)$. They should all have some functorial properties, summarized in the maxim (only a hope or guiding principle rather

than an established fact in this very general setting): quantization commutes with reduction.

To report on the recent progress overcoming a certain number of difficulties, arising in the case of a noncompact manifold, or a noncompact group G, was an important goal for this meeting. However, important results on the quantization in the case of compact manifolds were also reported in this meeting.

Thus the topic of our meeting included deformation quantization of functions on a symplectic manifold via Toeplitz operators, branching rules for unitary representations of real Lie groups, equivariant index of transversally elliptic operators, quantization of Hamiltonian manifolds with proper moment maps, group valued moment maps, Lagrangian fibrations.

It was not clear to the organizers that our choice of participants working on these many diverse topics and with many different techniques (topological K-theory, analytic estimates, C^*-algebras, representation theory) could lead to anything other than a series of talks with disjoint attendance. However, we think that the meeting was very successful in making bridges between the many different approaches towards a general common goal. This is certainly due to the very unique atmosphere of the Oberwolfach setting.

There were 22 talks of approximately 50 minutes, 7 talks of 30 minutes, and a session of short talks by young postdocs. All the speakers presented interesting new results, and they were concerned with clearly communicating the results of their research to an audience, possibly not familiar with techniques used, although interested in same themes. Thus our meeting was successful due to the efforts of the speakers. Certainly, this meeting will produce new ideas in the future in the participants' research, generated by attending a live presentation of new points of view.

Let us give some details on the topics of the conference:

• Quantization $Q(M, L)$ of a noncompact symplectic manifold M provided with a line bundle L.

Methods via C^*-algebras, or transversally elliptic operators or cutting methods were presented.

Hamiltonian manifolds such as the cotangent bundle of a manifold are a classical topic in mechanics. The list of other interesting Hamiltonian manifolds include the coadjoint orbits of real reductive Lie groups, representation spaces of fundamental groups of a surface of genus g with value in a compact Lie group (moduli spaces of flat bundles) or in a complex lie group (Hitchin moduli spaces). Results on the quantisation of those manifolds were discussed.

• Toeplitz algebras: this leads to quantisation of the algebra of functions on a symplectic manifold by studying asymptotic k estimates of the quantisation $Q(M, L^k)$ when the line bundle L is raised to its k-th power.

• The equivariant index of elliptic operators or of transversally elliptic operators.

Methods via C^*-algebras, the heat kernel or topological K-theory were presented.

• Quantisation of integrable systems. This includes the theory of Lagrangian fibrations (possibly singular) and Bohr-Sommerfeld orbits.

• Spectrum of the Laplacian or the hypoelliptic Laplacian. Zeta functions of the Laplacian, analytic torsion.

Detailed information on the topics presented are given in the abstracts.

We had asked several young researchers to prepare a poster on their research before coming to Oberwolfach. Wednesday evening was then devoted to a special session of short talks and posters. Talks given by the younger researchers were dynamic and very well prepared. Furthermore, although the talks were necessarily very short due to the lack of time, we had a poster session just after the introductory talks, and scientific informal discussions. This was the "must-see event" of the workshop, and it went very well.

On behalf of all participants, we would like to thank the staff for their concern in providing the best material conditions for our stay. The setting of Oberwolfach is beautiful, the food of excellent quality, the library full of resources, and the staff extremely helpful.

Thanks to Oberwolfach grants, four young researchers: Solha (Barcelona), Deltour (Montpellier), Hochs (Utrecht), Szilagyi (Geneve) could participate to our workshop.

Finally, as organizers, we would like to thank the director and his staff for their great help in the scientific organization. In particular, the director explained the general policy of the Oberwolfach meeting to us, which was very helpful.

1. Program of the conference

Monday, 14/02/2011

9h00-10h00 G. Marinescu
Toeplitz operators and geometric quantization

10h10–11h10 P. Ramacher
Singular equivariant heat asymptotics and Lefchetz formulas

11h20 -12h20 M. Duflo
Kirillov's formula and Box splines

14h30–15h30 P-E. Paradan
Spin quantization in the compact and non-compact setting

16h00-17h00 T. Kobayashi
Geometric quantization, limits and restrictions–some examples for elliptic and minimal orbits

17h10-18h10 B. Ørsted

Deformation of Fourier transformation

Tuesday, 15/02/2011

9h00-10h00 J. Brüning
Formulas for the multiplicities of the equivariant index

10h10–11h10 W. Zhang
Geometric quantisation for proper moment maps

11h20 -12h20 L. Boutet de Monvel
Asymptotic equivariant index of Toeplitz operators on spheres

16h00-17h00 W. Müller
Dynamical zeta functions and analytic torsion

17h10-18h10 K-I. Yoshikawa
Singularities and Analytic Torsion

Evening talk: 20h00-21h00 J.M. Bismut
Hypoelliptic Laplacian

Wednesday, 16/02/2011

8h50-9h40 G. Kasparov
K-theoretic index theorems for transversally elliptic operators

9h50-10h40 V. Mathai
Geometric quantization commutes with reduction

11h00–11h50 P. Piazza
Eta cocycles, relative pairings and Godbillon-Vey index theorem

12h00 -12h50 T. Schick
L^2-Betti numbers and their values

20h00 –21h30 Short talks and Poster session
M. Hamilton
Real and complex quantization of flag manifolds
P. Hochs
Quantization commutes with reduction at non-trivial representation
S. Fitzpatrick
Quantization of manifolds with f-structure
R. Solha

Geometric quantization of integrable systems with singularities
Z. Szilagyi
An equivariant Jeffrey-Kirwan formula in non-compact case

Thursday, 17/02/2011

9h00-10h00 H. Fujita
Localization of Riemann-Roch numbers via Torus fibrations

10h10–11h10 A. Szenes
Enumerative topology of quotients

11h20 -12h20 E. Meinrenken
Verlinde formulas for non-simply connected groups

16h00-17h00 G. Heckman
On the regularization of the Kepler problem

17h10- 17h40 G. Deltour
Symplectic and Hamiltonian properties of holomorphic co-adjoint orbits

17h40– 18h10 A-L. Mare
On the image of real loci of symplectic manifolds under moment maps

Friday, 18/02/2011

9h00-9h30 E. Miranda
From action-angle coordinates to geometric quantization: a 30-minute round trip

9h30-10h00 C. Procesi (presented by M. Vergne)
Multiplicities formulas for transversally elliptic operators

10h20-10h50 S. Wu
Quantization of the cotangent bundle of Lie groups

10h50–11h20 J. Huebschmann
Singular Kähler quantization on the moduli space of semi- stable holomorphic vector bundles on a curve

11h40– 12h10 S. Goette
Perturbative analysis of the L^2 heat kernel for large times

13h50–14h50 A. Alekseev
Tropical avatar of the Gelfand-Zeitlin integrable system