

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Topological and Geometric Combinatorics

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ABSTRACT. The 2011 Oberwolfach meeting “Topological and Geometric Combinatorics” was organized by Anders Björner (Stockholm), Gil Kalai (Jerusalem), Isabella Novik (Seattle), and Günter M. Ziegler (Berlin). It covered a wide variety of aspects of Discrete Geometry, Topological Combinatorics, and Geometric Topology. Some of the highlights of the conference included (1) counterexamples to the Hirsch conjecture, (2) the latest results around the colored Tverberg theorem, and (3) recent developments on the complexity of the simplex algorithm.

Mathematics Subject Classification (2000): 05xx, 52xx, 55xx, 57xx.

Introduction by the Organisers

The 2011 Oberwolfach meeting “Topological and Geometric Combinatorics” was organized by Anders Björner (KTH and Mittag-Leffler Institute, Stockholm), Gil Kalai (Hebrew University, Jerusalem), Isabella Novik (University of Washington, Seattle), and Günter M. Ziegler (Technical University, Berlin). The conference consisted of three one-hour lectures by Francisco Santos, Pavle Blagojević, and Thomas Hansen on outstanding recent developments in the field, as well as twenty-eight talks ranging from half-hour to 45-minute presentations, a problem session (led by Gil Kalai), and many more informal sessions, group discussions, and a great variety of small group and pairwise discussions. It was a very productive and enjoyable week.

The conference treated a broad spectrum of topics from Discrete Geometry (such as polytopes, epsilon nets, rigidity, complexity, etc.), Topological Combinatorics (such as problems surrounding Tverberg's theorem, topological representations of matroids, poset topology, etc.), and Geometric Topology (triangulated manifolds, embeddings of polyhedra, homology of random complexes, etc.). It is impossible to summarize in one-page report the richness and depth of the work and presentations. Instead we will concentrate here on some of the highlights.

The very first lecture on Monday was given by Francisco Santos and was devoted to describing his recent counterexamples to the Hirsch conjecture. This 53-year-old conjecture posits that the diameter of a convex polytope with n facets in dimension d is at most $n - d$. Aside from its mathematical interest, this conjecture is important because the simplex method for solving a linear program walks along a path on the surface of a polytope (often in dimensions as large as $d = 10000$). Thus the diameter of the polytope provides a lower bound on its worst-case complexity, and establishing an upper bound on the diameter raises that complexity lower bound. In his talk Santos also announced the explicit polymake computation of an example (joint with Christophe Weibel): it is a 20-dimensional simple polytope with 40 facets and 36'442 vertices, of diameter 21.

Spectacular recent developments on the complexity of the simplex algorithm were presented in the talk by Thomas Dueholm Hansen describing his joint work with Oliver Friedmann and Uri Zwick. The three of them have managed to prove subexponential lower bounds of the form 2^{n^α} for two basic randomized pivot rules for the simplex algorithm. This is the first result of its kind and deciding if this is possible was an open problem for several decades.

A very impressive account on how topological methods were used in a very recent series of works by Pavle Blagojević, Benjamin Matschke, and Günter M. Ziegler to solve several long-standing problems surrounding Tverberg's theorem was given by Pavle Blagojević.

Janos Pach talked about his recent paper with Gabor Tardos describing a breakthrough in geometric constructions for ϵ -nets. It follows from the general theory of VC-dimension that *epsilon*-nets of size $O(\epsilon \log(1/\epsilon))$ can be constructed for various classes of geometric objects such as half spaces in Euclidean spaces. Pach and Tardos showed that this estimate is sharp for very simple geometric objects such as half spaces in four dimensions.

Patricia Hersh described new topological methods to study stratified spaces leading to a proof of a conjecture by Fomin and Shapiro on certain complexes arising from Coxeter groups. Mark Noy's lecture outlined the remarkable recent understanding of diameter of random planar triangulations with n vertices which in agreement with old mysterious conjectures from physics behaves like $n^{1/4}$.

We also cannot avoid mentioning a lively and incredible problem session: a large number of the problems/questions raised were answered on spot.

The collection of abstracts below presents an overview of the official program of the conference. It does not cover all the additional smaller presentations, group discussions and blackboard meetings, nor the lively interactions that occurred

during the week. However, it does convey the manifold connections between the themes of the conference, refinements of well-established bridges, completely new links between seemingly distant themes, problems, methods, and theories, as well as demonstrates substantial progress on older problems. In short, it shows that the area is very much alive!

We are extremely grateful to the Oberwolfach institute, its director and to all of its staff for providing a perfect setting for an inspiring, intensive week of “Topological and Geometric Combinatorics”.

Anders Björner, Gil Kalai, Isabella Novik, Günter M. Ziegler
Stockholm/Jerusalem/Seattle/Berlin, March 2011

