Mathematisches Forschungsinstitut Oberwolfach

Report No. 07/2011

DOI: 10.4171/OWR/2011/07

Trends in Mathematical Imaging and Surface Processing

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January 30th – February 5th, 2011

Abstract. Motivated both by industrial applications and the challenge of new problems, one observes an increasing interest in the field of image and surface processing over the last years. It has become clear that even though the applications areas differ significantly the methodological overlap is enormous. Even if contributions to the field come from almost any discipline in mathematics, a major role is played by partial differential equations and in particular by geometric and variational modeling and by their numerical counterparts. The aim of the workshop was to gather a group of leading experts coming from mathematics, engineering and computer graphics to cover the main developments.

Mathematics Subject Classification (2000): 35Q80, 49Q10, 49Q20, 65D18, 65K10, 65M60, 65M32, 68U10, 68U05.

Introduction by the Organisers

Variational methods and partial differential equations form a core of various methodologies in image analysis, computer vision and surface processing. *Geometric concepts* naturally appear in the modeling, processing or animation of surfaces for instance in computer graphics. Furthermore, the space of shapes - either explicitly given as parametric surfaces or implicitly encoded in 2D and 3D images - has an interesting geometric structure, the understanding of which is crucial for many applications. This workshop brought together researchers from computer vision as well as computer graphics and mathematicians with expertise in the calculus of variations, in PDE analysis and numerics, in discrete geometry and sparse approximation. The aim of the workshop was to dovetail the strengths of geometry,

analysis and numerics in order to get insight into new models.

Different from the previous workshop in 2007 with a particular focus on image decomposition, different types of surface representation, feature preserving methods, and the proper discretization of associated PDE models, this time the perspective was more global. In fact, instead on the processing of single geometries or shapes the focus was more on the geometry of the space of shapes, or instead of continuous energy descent methods, relaxation techniques and associated global optimization approaches have been presented. Some areas of particular relevance in the workshop have been

- *Shape space analysis*, with the rigorous definition of shape space as a Riemannian manifold, the efficient computation of the geodesic distance, the Fréchet or Karcher mean, the principal component analysis of shapes, a suitable embedding into the context of shape statistics, and different types of underlying distances such as the Wasserstein distance, a distanced based on viscous dissipation along motion paths, or the Gromov Hausdorff distance,

- *Global optimization approaches* in image labeling or segmentation, image impainting, and motion estimation, where convex relaxation is applied in the context of TV and L^1 minimization problems and the appropriate use of primal and dual optimization strategies leads to highly efficient algorithms,

- *Concepts from discrete geometry* and the modeling of discrete free form surfaces in architecture, where methods from discrete exterior calculus lead to effective discretization and fabrication restrictions on glass roofs pose challenging design problems and give new impulses to discrete geometry and integrable systems.

Let us finally mention that in monday afternoon we had a special get–to–know session with 3 minute statements of almost all participants on open problems, new methodologies, and demanding applications prepared the ground for intensive discussion between the participants from diverse disciplines over the whole week.