

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Stochastic Analysis in Finance and Insurance

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January 23rd – January 29th, 2011

ABSTRACT. This workshop brought together leading experts and a large number of younger researchers in stochastic analysis and mathematical finance from all over the world. During a highly intense week, participants exchanged during talks and discussions many ideas and laid foundations for new collaborations and further developments in the field.

Mathematics Subject Classification (2000): 60Gxx, 60Hxx, 91Bxx.

Introduction by the Organisers

The workshop *Stochastic Analysis in Finance and Insurance*, organised by Dmitry Kramkov (Pittsburgh), Martin Schweizer (Zürich) and Nizar Touzi (Paris) was held January 23rd – January 29th, 2011. The meeting had a total of 53 participants from all over the world with a deliberately chosen mix of more experienced researchers and many younger participants.

During the five days, there were a total of 24 talks with many lively interactions and discussions. In addition, there were a historical lecture and two blocks of short communications, as will be explained below.

The topics presented in the talks covered a very wide spectrum. Major developments included a focus on new statistical problems, new mathematical and modelling issues arising out of and in connection with the recent financial crisis, and as always a number of foundational questions. To stimulate discussions and maximise interactions, talks were deliberately not organised into groups by major topics. A short overview of the talks given day by day looks as follows.

Philip Protter in the first talk of the workshop presented ideas on how one could discover financial bubbles in real time, combining ideas from local martingale modelling with statistical tools. *Marcel Nutz* presented new results on G -expectations in order to study markets with uncertainty about the volatility of assets. *Jean Jacod* gave an overview of recent developments in statistical problems for financial data and highlighted the difficulties coming from jumps in prices. *Christian Bender* introduced the concept of simple arbitrage with the goal of enlarging the class of feasible models by reducing arbitrage conditions to practically realistic assumptions. *Matheus Grasselli* presented a mathematical description of a model introduced by the economist Hyman Minsky in order to explain asset price bubbles from basic economic considerations. Finally, *Sergey Nadtochiy* explained the ideas behind forward performance processes to model optimal investment behaviour and showed in a class of examples how this leads to ill-posed Hamilton–Jacobi–Bellman equations.

Albert Shiryaev started the second day with an example of a non-classical testing problem for Brownian motion with drift, involving three instead of the usual two hypotheses. *Christoph Frei* gave examples of multidimensional quadratic backward stochastic differential equations having (in contrast to the one-dimensional case) no solution, and explained how these equations come up and can be used in connection with equilibrium problems in financial markets. *Peter Tankov* presented limit results for time-changed Lévy processes sampled at hitting times, instead of at fixed times, and showed how these can be used in a financial context. *Christoph Czichowsky* gave a new formulation for the classical Markowitz problem to overcome the well-known time-inconsistency problems associated with that criterion, and showed by relating discrete- and continuous-time theory that the new formulation is both natural and mathematically interesting. *Ronnie Sircar* used stochastic differential games and the associated Hamilton–Jacobi–Bellman equations to discuss the approaches by Bertrand and Cournot to study oligopolistic markets. At the end of the day, *Roger Lee* presented an effective mechanism to generate asymptotic expansions of arbitrarily high order for implied volatility.

On Wednesday, *David Hobson* presented new model-independent bounds for variance swaps with the help of Skorokhod embedding results. *Johannes Muhle-Karbe* gave new asymptotic results for portfolio optimisation with transaction costs by exploiting the recently developed idea of shadow prices. In addition, there were a number of short communications in a newly introduced format. Each presenter had 5 minutes to explain his result, which were then followed by 5 minutes of questions and discussion. This idea of explaining in a nutshell some current problems or results met with enormous success; the list of volunteers for giving a short presentation very quickly grew to a total of 17 names, and the corresponding talks were scheduled on Wednesday morning and Thursday morning. Wednesday afternoon was then reserved for the traditional excursion, which went to Oberwolfach Kirche instead of St. Roman because there was still quite a lot of snow and many tracks on the hills were very slippery.

Thursday started with *Luciano Campi* who presented a structural model for pricing and hedging derivatives in energy markets, a topic of increasing practical importance in recent years. *Jin Ma* used a system of interacting stochastic differential equations to describe possible defaults of correlated assets, and proved a law of large numbers for self-exciting dynamics via a fixed-point argument. A second block of short communications followed, leading again to intense discussions that continued into the afternoon and in the evenings. *Mete Soner* then gave new existence and uniqueness results for second order backward stochastic differential equations, a probabilistic analogue to a class of fully nonlinear partial differential equations. *Kasper Larsen* showed how a number of asset pricing puzzles from finance can be explained, via a clever construction, by equilibria in Brownian-driven but incomplete financial markets. Finally, *Albert Shiryaev* gave a historical talk in memory of the recently deceased Anatoli V. Skorokhod, one of the great Russian probabilists born in the 20th century.

On the last day, *Tahir Choulli* presented new ideas and results in connection with defaultable markets; in mathematical terms, this amounts to studying the behaviour of stochastic processes before and after a random time, and this leads to some quite challenging new problems. *Josef Teichmann* discussed affine processes and their applications in mathematical finance, focusing in particular on regularity and filtering questions. Complementing an earlier talk, *Peter Friz* derived new expansion results for the Heston model, one of the workhorses in practical applications of option pricing. *Jan Obloj* studied the inverse problem of recovering the preferences of financial agents from their observed actions and showed that uniqueness as well as nonuniqueness can happen, depending on the setting. *Mihai Sîrbu* introduced a model for high-watermark fees in hedge fund investments and explained how to fruitfully use the Skorokhod equation in that context. Finally, *Freddy Delbaen* gave a new, more structural proof for the representation of the penalty function in time-consistent monetary utilities.

Like in the workshop three years before, there were an enormous number of discussions, interactions and exchanges. Everyone felt privileged to be able to spend a highly productive and creative week at the unique place that has been created in Oberwolfach and to profit from the excellent infrastructure, support and scientific environment. In particular, the younger participants and first-time visitors to Oberwolfach unanimously said that the actual experience of the workshop and the overall scientific atmosphere still exceeded their already high anticipations.

As organisers and on behalf of all participants, we want to express our gratitude to the Mathematisches Forschungsinstitut Oberwolfach for giving us the opportunity of having this very successful workshop, and we hope that we shall be able to come back at some time in the future.

Dmitry Kramkov
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