

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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**Mini-Workshop: Nonlinear Least Squares in Shape
Identification Problems**

Organised by
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January 16th – January 22nd, 2011

ABSTRACT. This mini-workshop brought together mathematicians engaged in shape optimization and in inverse problems in order to address a specific class of problems on the reconstruction of geometries. Such a problem can be formulated as an inverse problem forgetting the shape point of view or as minimization with respect to the shape.

Mathematics Subject Classification (2000): 49N45, 65N21.

Introduction by the Organisers

The workshop *Nonlinear Least Squares in Shape Identification Problems*, organised by Marc Dambrine (Pau), Frank Hettlich (Karlsruhe) and Roland Potthast (Berkshire) was held January 16th–January 22nd, 2011. This meeting was rather well attended with over 14 participants with strong european representation since it suffered late cancellations caused by the events in Tunisia. This workshop was a nice blend of researchers with various backgrounds and generated fruitful scientific discussions mixing different points of view.

The typical question addressed within the workshop is the inverse problem of determining an inclusion from measurements on the boundary of a domain. To that end, many strategies have been developed. Some approaches are based on shape optimization: for a guessed inclusion, considered as the variable, is associated a cost related to the measurements that has to be minimized. Some approaches are direct like the factorization method: a criterion is defined to check if a point belongs or not to the unknown inclusion. However, the problem is severely ill-posed: the unknown does not depend continuously on the measurements. Hence, every

method has to face this difficulty and requires adapted regularization methods in order to find a numerical approximated solution.

The mini-workshop aimed at mixing people from different backgrounds using various methods to address the question of shape identification in order to discuss the connections between the approaches. Hence, the given talks can be grouped in three main themes:

- iterative methods based on a shape calculus for the detection of objects,
- direct methods for the detection,
- regularization methods for this inverse problem.

The first theme was covered by the presentations of Marc Dambrine, Eric Darri-grand, Helmut Harbrecht, Frank Hettlich, Barbara Kaltenbacher and Maria-Luisa Rapun. Furthermore, Corinna Burkard, Thomas Fiedler, Andreas Kirsch, Gen Nakamura and Roland Potthast were the speakers for the second theme. Finally, Barbara Kaltenbacher, Armin Lechleiter and Andreas Rieder gave talks on the last theme. Frédérique Le Louër gave a talk mixing both the first and third themes.

Some free discussion meetings were organized that allows the meeting participants to gather some open questions that seem of importance to the group and could be new research directions. For example, we discussed and selected among others the following questions:

- of the connections between the Radon transform of characteristic functions and the differentiation with respect to the domain,
- of recovering time-dependent objects; of shape derivatives for parabolic problems and for transmission problems,
- of obtaining convergence theories for dynamic Tikhonov regularization and for inexact Newton method under more reasonable assumptions,
- of the definition of resolution in electrical impedance tomography or in inverse scattering.

The organizers and participants of the mini-workshop are grateful to the *Mathematisches Forschungsinstitut Oberwolfach* for providing a very pleasant and inspiring setting for the workshop allowing us to focus on the mathematical questions of importance.