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Combinatorics

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Abstract. Combinatorics is a fundamental mathematical discipline which focuses on the study of discrete objects and their properties. The current workshop brought together researchers from diverse fields such as Extremal and Probabilistic Combinatorics, Discrete Geometry, Graph theory, Combiantorial Optimization and Algebraic Combinatorics for a fruitful interaction. New results, methods and developments and future challenges were discussed. This is a report on the meeting containing abstracts of the presentations and a summary of the problem session.

Mathematics Subject Classification (2000): 05Cxx, 05Dxx, 52Bxx, 68Rxx.

Introduction by the Organisers

The workshop Combinatorics organised by Jeff Kahn (Piscataway), Angelika Steger (Zürich), and Benny Sudakov (Los Angeles) was held January 2nd - January 8th, 2011. Despite the early point in the year the meeting was extremely well attended with 52 participants from the US, Canada, Brazil, UK, Israel, and various European countries. The program consisted of 13 plenary lectures, accompanied by 19 shorter contributions and a vivid problem session led by Vera T. Sós. The plenary lectures were intended to provide overviews of the state of the art in various combinatorial areas and/or in-depth treatments of major new results. The short talks ranged over a wide variety of topics including in graph theory, coding theory, discrete geometry, extremal combinatorics, Ramsey theory, theoretical computer science, and probabilistic combinatorics. Special attention was paid throughout to providing a platform for younger researchers to present themselves and their results.

This report contains extended abstracts of the talks and the statements of the problems that were posed during the problem session. This was a particularly successful edition of the meeting Combinatorics, in large part because of the exceptional strength and range of the results discussed. Here we mention just a few of these, each of which involved spectacular progress on some well-known, longstanding conjecture. These few snapshots also provide a nice, if small, sample of the large variety of topics and methodologies that were presented during a fascinating week in Oberwolfach.

A family of graphs $\mathcal{F}$ is triangle-intersecting if for every $G, H \in \mathcal{F}$, the intersection $G \cap H$ contains a triangle. A celebrated conjecture of Simonovits and Sós from 1976 states that the largest triangle-intersecting families of graphs on a fixed set of n vertices are those obtained by fixing a specific triangle and taking all graphs containing it. This conjecture was recently proved by Ellis, Filmus, and Friedgut using spectral methods and discrete Fourier analysis; see the abstract of Ehud Friedgut's talk.

For a graph G, write $\Phi(G)$ for the number of perfect matchings of G. The Lovász-Plummer Conjecture (proposed in the 1970's) says that for bridgeless, cubic graphs G, $\Phi(G)$ grows exponentially in the number (say n) of vertices of G — this despite the fact that until two years ago it was not even known that the number must be more than *linear* in n. Not long before the Oberwolfach meeting, the Lovász-Plummer Conjecture was proved in full by Esperet, Kardoš, King, Král, and Norine, using a combination of ideas from graph theory and linear programming; see the Daniel Král's abstract.

A tournament is an orientation of a complete graph. Sumner's Universal Tournament Conjecture of 1971 says that any tournament on $2n-2$ vertices contains every directed n-vertex tree. Following a long history of partial results, Sumner's conjecture has now been proved in full, assuming only that n is large, by Kühn, Mycroft and Osthus, using, *inter alia*, probabilistic ideas and a variant of Szemerédi's Regularity Lemma; see the abstract of Deryk Osthus.

A subject that's received a great deal of attention over the last decade or two concerns questions of the type: given a (large) finite set Γ and family $\mathcal{F}$ of subsets of Γ that can be shown to exhibit some structural property of interest, about how large a random subset of Γ will (probably) continue to exhibit said property? The last few years have seen breakthrough progress on such questions by Mathias Schacht (see his abstract) and (independently and using very different methods) Conlon and Gowers. These results provide very general information on questions of the above type, including, to give just two examples, the threshold for Szemerédi's (arithmetic progressions) Theorem to hold in random subsets of the integers, and verification of a much-studied conjecture of Kohayakawa, Łuczak, and Rödl (1997) on Turán-type problems in random graphs.

On behalf of all participants, the organisers would like to thank the staff and the director of the Mathematisches Forschungsinstitut Oberwolfach for providing such a stimulating and inspiring atmosphere.