Abstract. Linear series have long played a central role in algebraic geometry. In recent years, starting with seminal papers by Demailly and Ein-Lazarsfeld, local properties of linear series – in particular local positivity, as measured by Seshadri constants – have come into focus. Interestingly, in their multi-point version they are closely related to the famous Nagata conjecture on plane curves. While a number of important basic results are available by now, there are still a large number of open questions and even completely open lines of research.

Mathematics Subject Classification (2000): 14C20, 14E25.

Introduction by the Organisers

The mini-workshop Linear Series on Algebraic Varieties gathered together experts in the subject with a spread of backgrounds and professional experience. Participants came from all over the world: US (Colorado, Illinois, Nebraska), Europe (France, Germany, Italy, Norway, Poland, Spain, Sweden) and Asia (Korea) and ranged from post docs to senior researchers. This variety of experience and background greatly contributed to generating stimulating discussions during the talks and the working group sessions, leading to what we believe will be the basis for several research collaborations.

The theme of the workshop

The theme of the workshop revolved around two related conjectures which in recent years stimulated important developments in the field of linear series, the
Nagata conjecture [19] and the SHGH conjecture [21, 11, 10, 14], sometimes also referred to as the Harbourne-Hirschowitz conjecture. The Nagata conjecture bounds the least degree $d$ of a curve passing through $r \geq 10$ general points in the projective plane with prescribed multiplicities $m_1, \ldots, m_r$. While this conjecture has remained open for half a century, several people observed recently that it could be a piece of a much more general picture, which is by no means specific to $\mathbb{P}^2$ ([3], [2]).

The concept which allows passing from a rather special question on the projective plane to a much more general setting is that of Seshadri constants. Recall that given a polarized variety $(X, L)$ and a subscheme $Z$ of $X$ the Seshadri constant of $L$ at $Z$ is the real number $\varepsilon(X, L; Z) := \sup \{ \lambda : f^* L - \lambda E \text{ is ample on } Y = \text{Bl}_Z X \}$ (see [2] for details). An elegant and uniform way to generalize the Nagata conjecture is to assert that if $Z$ is a union of sufficiently many and sufficiently general reduced points of $X$, then the Seshadri constant $\varepsilon(X, L; Z)$ is the maximal possible. Recently there were several interesting developments towards approximating from below the numbers $\varepsilon(X, L; Z)$ ([12], [9], [20], [5], [13], [15]). During the mini-workshop various approaches to this problem were presented.

The SHGH conjecture gives a precise prediction for the actual dimension of the linear series $L(d; m_1, \ldots, m_r)$ of curves of degree $d$ passing through $r$ general points of the plane with multiplicities $m_1, \ldots, m_r$, giving a simple geometric condition for a linear series of this type to be special, i.e., of higher dimension than is expected by a naive dimension count. Several approaches have been developed to attack this problem. We first mention the so-called Horace method, a specialization method introduced by Alexander and Hirschowitz [1] and pursued by Mignon [18], Roé [20], and Evain [9]. A second approach is Ciliberto and Miranda’s [4, 5, 6] method of degenerations of the underlying variety for linear series on families of planes. A third approach relates the problem to packing type questions in symplectic geometry as observed by McDuff-Polterovich [17] and pursued by Biran [3] and recently by Eckl [8]. Finally there are approaches via symbolic computations, most prominently those of Lorentz-Lorentz [16] and Dumnicki [7].

**The structure of the workshop**

The aim of the workshop was twofold: to gather together experts working on the three different aspects mentioned above, and to stimulate collaboration by discussing open problems in the field. For this reason every day consisted of two main activities:

- research talks, typically two or three in the morning; and
- working group discussions, in the afternoon.

A list of possible questions to work on during the workshop was discussed via email well ahead of the workshop. During the first day, the final selections were made. Two main areas of interest emerged from the discussion: asymptotic approaches and combinatorial ones. Consequently two working groups were formed. The workshop was just the starting point and an ignition to collaborate on the chosen problem. The working groups continue their efforts. The outcome of these discussions will appear elsewhere.
REFERENCES
