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Topologie

Organised by
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Abstract. This conference is one of the few occasions where researchers from many different areas in algebraic and geometric topology are able to meet and exchange ideas. Accordingly, the program covered a wide range of new developments in such fields as geometric group theory, rigidity of group actions, knot theory, and stable and unstable homotopy theory. More specifically, we discussed progress on problems such as the Farrell-Jones conjecture, the Levine conjecture in grope cobordism of knots and Rosenberg's conjecture about homotopy invariance of negative algebraic K-theory, to mention just a few subjects with a name attached. One of the highlights was a series of four talks on the solution of Arf-Kervaire invariant problem by Mike Hill and Doug Ravenel, reporting on their joint work with Mike Hopkins.

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Introduction by the Organisers

This conference was the first topology conferences in Oberwolfach organized by Thomas Schick, Peter Teichner, Nathalie Wahl and Michael Weiss. About 50 mathematicians participated, working in many different areas of algebraic and geometric topology.

The 18 regular talks of the conference covered a wide range of areas such as 3-manifolds and knot theory, geometric group theory, algebraic K- and L-theory, and homotopy theory. One of the goals of the conference is to foster interaction between such different areas and the passage of methods from one to the other. Four of these talks were devoted to the solution of the Kervaire invariant question by Mike Hopkins, Mike Hill and Doug Ravenel, allowing an in-depth discussion of the new ideas necessary for this breakthrough on a decades-old problem in homotopy theory.

In addition, to give the many young and very young participants the opportunity to present themselves and their work to a broader audience, a "gong show" was organized where eight participants gave an overview on their research efforts and results. Here, **Alexander Kahle** from Göttingen reported on joint work with Alessandro Valentino concerning T-duality and differential K-theory. In this work, topological T-duality (studied for example by Bunke-Schick or Mathai-Rosenberg) is enriched with geometric information using differential K-theory, and indeed a T-duality isomorphism for differential K-theory is established for pairs which are T-duals of each other. **Arturo Prat-Waldron** from Berkeley discussed Thom classes for field theories. The space of certain low dimensional field theories is used as a geometric model for associated generalized cohomology theories. The goal now is to find a geometric description of Thom classes in this model. **Ulrich Pennig** from Münster reported on twisted K-theory and obstructions to positive scalar curvature. In his work, he gives a new geometric model for twisted K-theory, proves an index theorem in the spirit of Kasparov's in this context and uses this to extend the enlargeability-obstruction to positive scalar curvature to non-spin manifolds. **Georgios Raptis** from Osnabrück described K-theory of derivators. He gives an example of two differential graded algebras that have the same derivator K-theory but non-isomorphic Waldhausen K-theory. He also proves that Maltsiniotis' comparison and localization conjectures for derivator K-theory cannot be simultaneously true. **Justin Noel** from Strasbourg studied the complex orientations preserving power operations. This is used in particular to classify which complex oriented cohomology theories can be given an H-infinity ring structure compatible with the standard E-infinity ring structure on MU. **Lennart Meier** from Bonn gave a new proof, from a stacky point of view, of Bousfield's classification of modules over real K-theory KO. The crucial ingredient is to find all KO-modules M such that $M \wedge_{KO} KU$ is KU-free. This approach should eventually lead to a generalization to modules over topological modular forms TMF. **Wolfgang Steimle** from Münster described obstructions to stably fibering manifolds. In particular, he introduced and calculated family versions of Whitehead torsion,

using higher algebraic K-theory of spaces. **David Ayala** from Copenhagen described a simple and combinatorial E_n operad which is built out of finite posets indexing a stratification of configuration spaces of points in an n-disk, posets which recently became important for modeling weak n-categories. A version of Dunn's additivity theorem is a formal consequence of the set up.

We now report on some of the highlights of the regular talks, whose abstracts form the main part of this report.

Arthur Bartels talked about joint work with Wolfgang Lück and Tom Farrell, proving the Farrell-Jones conjecture and algebraic K-theory conjecture for cocompact lattices in connected Lie groups. One of the most spectacular consequences is the Borel conjecture on topological rigidity of aspherical manifolds whose fundamental group is a cocompact lattice as above. The proof uses controlled topology and new constructions of transfer homomorphisms. In other talks on geometric group theory, Michelle Bucher-Karlsson described the explicit calculation of the Gromov norm of the universal Euler class in the cohomology of classifying space for oriented vector bundles, and Stefan Friedl gave a survey on the group theoretic properties of fundamental groups of three manifolds, culminating in his joint result with Matthias Aschenbrenner that all such groups are for every prime number p virtually residually p.

Andreas Thom presented his joint work with Guillermo Cortiñas that negative algebraic K-theory of the algebra $C(X)$ is a homotopy invariant of the space X. This finally implements a strategy proposed several decades ago by Jonathan Rosenberg. Among many other tools, algebraic geometry —including Hironaka's resolution of singularities— is crucially used.

Cameron Gordon gave (in joint work with Danny Calegari) a complete classification of all knots in closed 3-manifolds of small rational genus. Jim Conant presented a proof of the Levine conjecture and applications to grope cobordism (this is joint work with Peter Teichner).

An curious application of homotopy theory to the theory of finite groups was described by Bob Oliver. He presented a purely algebraic result about the p-subgroup structure in finite groups of Lie type, one which was reduced to a statement about their classifying spaces and then proved using the homotopy theory of p-local finite groups. No algebraic proof of this result is available up to now. Jesper Grodal used homotopy theory to study the group of self-homotopy equivalences of a finite CW-complex X and found explicit bounds on the order of groups with a faithful action up to homotopy on X.

Bernhard Hanke described a new construction of families of manifolds with positive scalar curvature, jointly carried out with Boris Botvinnik, Thomas Schick and Mark Walsh, which is based on a family version of the Gromov-Lawson surgery method and which provides the first examples of non-trivial elements of the moduli space of metrics of positive scalar curvature.

Other talks addressed a construction of a delooping of the 2-category of von Neumann algebras (with bimodules and Connes fusion as 1-morphism and 2-morphism) via conformal nets, the complete classification of characteristic cohomology classes of Morita-Miller-Mumford type for bundles of closed manifolds of arbitrary dimension, stable moduli spaces of highly connected high-dimensional manifolds, and algebraic models of equivariant stable homotopy theories.

The famous Oberwolfach atmosphere made this meeting another wonderful success, and all thanks go to the institute for creating this atmosphere and making the conference possible.