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Non-positive Curvature and Geometric Structures in Group Theory

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Abstract. The focus of this meeting was the use of geometric methods to study infinite discrete groups. Key topics included isometric actions of such groups on spaces of nonpositive curvature, such as CAT(0) cube complexes, buildings, and hyperbolic or symmetric spaces. These actions lead to a rich and fruitful interplay between geometry and group theoretic questions.

Mathematics Subject Classification (2000): 20Fxx, 57Mxx.

Introduction by the Organisers

The meeting focused on several areas of current excitement in geometric group theory, unified by the important role that non-positive curvature plays in each. The geometric approach to group theory dominates the modern study of finitely generated groups. A central idea in this approach is that one illuminates the nature of groups by studying their actions on spaces with appropriate geometric structure. The quality of information one gleans from the action depends on the richness of the geometric structure and the quality of the action (discrete and cocompact by isometries being the most desirable). A powerful illustration of this is provided by the study of isometric actions on spaces of non-positive curvature. The curvature hypothesis alone tells one a great deal about the algebraic structure of the group, but the theory becomes much richer when one imposes further hypotheses on the space. Prime illustrations of this are the the theory of buildings (J. Tits) and, most classically, the actions of discrete subgroups of semi-simple Lie groups on Riemannian symmetric spaces (É. Cartan).

The topics covered during this workshop can each be seen as a natural extension of an aspect of this last beautiful subject: rigidity, fixed point theorems, questions of linearity and residual finiteness, analysis at infinity, cohomological issues, *etc.* The diverse techniques involved in the topics that emerge under these headings typically lie far from these classical origins, and the spaces that arise are typically highly singular — buildings, CAT(0) cube complexes, asymptotic cones, the curve complex and other spaces related to Teichmüller space, Outer Space, *etc.* But the classical situation still provides a stimulating analogy.

This diversity within a common framework was widely reflected in the speakers of the workshop. We concentrated on specific topics that have seen recent exciting progress. These include: the study of new classes of buildings, of CAT(0) cube complexes, lattices in the isometry groups of the latter spaces and related embedding results; recent insights into the nature of mapping class groups of surfaces and automorphism groups of free groups; recent definitive results on the nature of the full isometry groups of CAT(0) spaces that admit parabolics; and the introduction of powerful new tools of an analytic nature. More details can be seen in the individual abstract below. We had 55 participants from a wide range of countries, and 23 lectures. In addition, there were two special sessions in the evening, with lectures by Arthur Bartels on the recent proof of the Farrell-Jones Conjecture for hyperbolic and CAT(0) groups and by Mark Sapir on conjugacy growth in groups.

The staff in Oberwolfach was—as always—extremely supportive and helpful. We are very grateful for the additional funding for five young PhD students and recent postdocs through Oberwolfach-Leibniz-Fellowships. In addition, there was one young student funded through the DMV Student's Conference. We think that this provided a great opportunity for these students.

We feel that the meeting was exciting and highly successful. The quality of all lectures was outstanding, and outside of lectures there was a constant buzz of intense mathematical conversations. We are confident that this conference will lead to both new and exciting mathematical results and to new collaborations.