

MATHEMATISCHES FORSCHUNGSIINSTITUT OBERWOLFACH

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Combinatorial Representation Theory

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ABSTRACT. The workshop brought together researchers from different fields in representation theory and algebraic combinatorics for a fruitful interaction. New results, methods and developments ranging from classical and modular representation theory, the theory of symmetric functions and Lie theory to cluster algebras and connections to physics and geometry were discussed.

Mathematics Subject Classification (2000): 05xx, 14xx, 16xx, 17xx, 20xx.

Introduction by the Organisers

The workshop on *Combinatorial Representation Theory*, was organised by Christine Bessenrodt (Hannover), Francesco Brenti (Roma), Alexander Kleshchev (Eugene) and Arun Ram (Melbourne). It was attended by 54 participants coming from Europe, North America, Japan and Australia. In the 23 long and 8 short talks – many given by young participants – fascinating new developments and significant progress on deep conjectures were presented. The schedule still left ample time for many discussions; in the ideal environment of the institute there was a lively exchange of ideas, also with researchers in the "Research in Pairs" program. Indeed, there were exciting questions and discoveries every evening, continuing cooperations as well as starting new joint research.

The scope of the meeting embraced representations coming from many directions: finite and algebraic groups and different types of algebras and Lie algebras. The fruitful focus point was the common interest in combinatorial aspects, dealt with by very different methods; in many cases, the representation theory of the symmetric groups and related groups or related algebras plays an important rôle.

A series of talks was dedicated to the recently discovered Khovanov-Lauda-Rouquier (KLR) algebras. These graded algebras defined for any Lie type have many important connections to Lie theory, categorification, and representation theory of symmetric groups and Hecke algebras. D. Hill spoke on his work with Melvin and Mondragon on the classification of irreducible modules over KLR algebras of finite type along the program initiated by Kleshchev and Ram. M. Vazirani presented her work with Lauda which yields another classification of irreducible modules over KLR algebras of arbitrary type via defining and identifying a crystal structure on them. A. Mathas talked on his work with Hu on a cellular structure on cyclotomic KLR algebras of finite and affine type A . In view of the work of Brundan and Kleshchev this transports into a completely new *graded* cellular structure on cyclotomic Hecke algebras, which promises to be very important.

Modular representation theory was also a focus point. S. Ariki spoke about modular branching rules for affine Hecke algebras of type A . This is related to the identification of various classifications of irreducible modules over affine Hecke algebras. N. Jacon spoke on canonical bases in higher level Fock spaces in connection with Ariki's categorification theorem and modular representation theory of cyclotomic Hecke algebras. M. Fayers reported on the recent progress concerning the classification of irreducible Specht modules over cyclotomic Hecke algebras.

His work with S. Khoroshkin on the representation theory of usual and twisted Yangians was described by M. Nazarov, while S. Goodwin spoke on his work with J. Brown on representations of finite W -algebras of classical types corresponding to special nilpotent orbits, generalizing work of Brundan and Kleshchev in type A .

Classical representation theory continues to be a central topic. O. Yacobi's talk had classical invariant theory as its core, and I. Gordon discussed problems from the invariant theory of complex reflection groups, whose solutions involved the representation theory of rational Cherednik algebras. C. Caselli introduced projective reflection groups and showed that Gelfand models and bases for their coinvariant algebras can be obtained via the combinatorics of a dual projective reflection group. Some of these results are new even for the Weyl groups of type D . V. Miemietz explained deep results with W. Turner extending the applications of the Dlab-Ringel construction from finite dimensional to infinite dimensional algebras. G. Malle talked on work with G. Navarro, explaining combinatorics underlying the confirmation of a recent conjectured character criterion of nilpotent blocks of finite groups for many quasi-simple groups, J. Comes spoke on the representation theory of Deligne's tensor category $\underline{\text{Rep}}(S_t)$ (for each $t \in \mathbb{C}$) related to the symmetric group S_t , when t is an integer, and G. Han explained techniques for discovering surprising new hook length formulas. A. Henderson spoke on enumerative results for nilpotent orbits in classical groups of type B or C which refine results by Lusztig.

In an important talk which stood on its own, P. Fiebig spoke on his work with Arakawa on blocks of representations of affine Kac-Moody algebras at the critical level. He explained how to relate such blocks to the blocks of the category of modules over an associated small quantum group.

Symmetric function theory continues to be a vibrant area of research. M. Yip explained how the alcove walk techniques from the proof of the Ram-Yip formula for expanding Macdonald polynomials in terms of monomial symmetric functions apply to provide a Littlewood-Richardson rule for Macdonald polynomials; this generalizes the classical rule for multiplying Weyl characters and recent results on the product of Hall-Littlewood symmetric functions. J. Haglund and C. Lenart covered several aspects of the relation between the Ram-Yip formula and the Haglund-Haiman-Loehr formula; their relations to expansions in terms of Demazure bases and quasisymmetric functions were discussed by Haglund and Yip. The theory of quasisymmetric functions has seen increasing importance in the last ten years, including applications to Macdonald polynomials and Kazhdan-Lusztig theory. As explained by S. van Willigenburg, the quasisymmetric Schur functions satisfy natural analogues of many results that hold for the classical Schur functions, providing a good justification for their name. This opens natural major lines of research. A. Schilling presented a Murnaghan-Nakayama rule for noncommutative k -Schur functions, a result which is new even in the commutative case.

Connections to mathematical physics and representations of affine Lie algebras appeared in several talks and blossomed in the talk of J. de Gier explaining how certain parabolic Kazhdan-Lusztig polynomials and Macdonald polynomial theories enter in the solutions of eigenvalue problems in statistical physics: specifically, fully packed loop models. M. Marietti presented a closed combinatorial formula for the parabolic Kazhdan-Lusztig polynomials of the tight quotients of the symmetric groups; this implies the known formula for the maximal quotients and relies on a new class of superpartitions with fermionic number one. The talk of C. Stroppel focused on the interplay between symmetric functions, geometry, mathematical physics (quantum cohomology, Bethe Ansatz and Verlinde formulas). J. Kujawa explained the notion of categorical dimension in ribbon categories and a suitable graphical calculus; these categories include categories of finite dimensional representations of groups, Lie algebras, superalgebras and quantum groups.

Deep connections to geometry appeared also in other talks. S. Gaussent explained some of the "compression" by using walks on the one-skeleton of an affine Tits building. T. Lam gave detailed information about the structure of the totally positive part of a loop group, and a mysterious similarity between his product constructions and the cluster algebra constructions of regular functions in the coordinate ring of the unipotent part of a Kac-Moody group which appeared in J. Schröer's talk was noticeable. Indeed, the cluster algebras introduced by Fomin and Zelevinsky are a very important and active research area. L. Williams, J. Schröer and D. Hernandez impressively used cluster algebra technology in the study of preprojective algebras, canonical bases, total positivity, moduli spaces of surfaces, and the finite dimensional representations of affine Lie algebras. A highlight was L. Williams' talk on a proof of the celebrated nonnegativity conjecture for cluster algebras arising from punctured surfaces. Her result gives a combinatorial interpretation of the coefficients of the Laurent monomials in the cluster variables, and includes the nonnegativity result for cluster algebras of finite type.

