Mathematisches Forschungsinstitut Oberwolfach

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C*-Algebren

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Abstract. The theory of C*-algebras plays a major rôle in many areas of modern mathematics, like Non-commutative Geometry, Dynamical Systems, Harmonic Analysis, and Topology, to name a few. The aim of the conference “C*-algebras” is to bring together experts from all those areas to provide a present day picture and to initiate new cooperations in this fast growing mathematical field.


Introduction by the Organisers

A (unital) C*-algebra is a non-commutative analogue of the space of continuous functions on a compact Hausdorff space. Since a compact Hausdorff space can be recovered as the maximal ideal space of the algebra of continuous functions, non-commutative C*-algebras represent virtual “non-commutative” spaces. The theory of C*-algebras goes back to work of Murray and von Neumann, who first studied a special variant now known as von Neumann algebras (which represent the non-commutative measure theory). The theory developed rapidly after some ground breaking work of Gelfand and Naimark in 1943. In the 70’s and 80’s of the last century, the point of view that non-commutative C*-algebras should be regarded as function spaces of “non-commutative” topological spaces became more and more a central theme of the theory. As a consequence, completely new areas in mathematics, like Non-commutative Geometry or Free Probability evolved and we now see that the theory of C*-algebras became a very active field with applications in and interactions with almost all areas of modern mathematics.
The aim of the workshop $C^*$-algebras, organized by Claire Anantharaman-Delaroche, Siegfried Echterhoff, Mikael Rørdam, and Dan Voiculescu, is to bring together leading researchers from basically all areas related to the field. This gives a unique opportunity to maintain a broad view on the subject and to create new cooperations between researchers with different background. Among the 52 participants was a good number of young researchers, some of them already on the top of the field. There have been 27 lectures presented at the workshop with topics ranging from classification of $C^*$-algebras, group actions on $C^*$-algebras, orbit equivalence of dynamical systems, subfactor theory of von Neumann algebras, $C^*$-algebras and logic, continuous fields of $C^*$-algebras, group $C^*$-algebras, $C^*$-algebras in Quantum Field Theory, $C^*$-algebras related to number theory, free probability, the relation between $C^*$-algebras and Harmonic Analysis, and many others.

The abstracts presented in this report clearly show that there has been very exciting progress in many of the above mentioned areas. Much of this progress comes from interactions with other fields of mathematics. As an example for this we want to mention the beautiful results of Farah and Weaver on some very old conjectures regarding possible structures of non-separabe $C^*$-algebras. They use methods from set theory and logic to give unexpected answers which show that existence or non-existence of certain phenomena very much depend on which set-theoretical axioms we assume to be true. Ergodic theory in interaction with operator algebras is another subject where major advances have been achieved, revealing new rigidity phenomena in the relations between groups, group actions and von Neumann algebras. Two very recent such spectacular advances are presented in the report: using Popa’s powerful deformation/rigidity techniques, Ioana shows that everything is remembered in case of Bernoulli actions of Kazhdan property (T) groups; in the same spirit, another remarkable result of Ioana, Popa and Vaes provides the very first examples of groups entirely determined by their von Neumann algebras. As one of the many other exciting results we also mention the recent progress of Christensen and coworkers on pertubation theory of $C^*$-algebras, in which they finish a long-term program by showing (among other things) that two nuclear $C^*$-algebras which are “close” in a certain metric sense must be isomorphic.

It is a pleasure for the organizers to thank all participants of the workshop for their beautiful lectures and fruitful discussions. We also want to use this opportunity to thank the Mathematisches Forschungsinstitut Oberwolfach for providing a very stimulating environment and strong support for organizing this conference. Special thanks also go to the very competent and helpful staff of the institute.