Abstract. Highlights of this workshop on structural graph theory included new developments on graph and matroid minors, continuous structures arising as limits of finite graphs, and new approaches to higher graph connectivity via tree structures.

Mathematics Subject Classification (2000): 05Cxx.

Introduction by the Organisers

The aim of this workshop was to offer an exchange forum for the leading researchers from the various fields of structural graph theory. When preparing the invitations, we did not define any particular foci this time, but concentrated on people. All the same, some particularly active fields can be identified, and the workshop even brought some surprises as to what these are.

One area of strong recent activity are graph limits: properties of finite graphs are studied through a non-discrete limit objects they define. The idea behind this is that one limit object can encapsulate the typical features of the (finite) graphs with a given property, and methods from other areas of mathematics, both algebraic and analytical, can be brought to bear on them in a way alien to individual discrete structures. This has become an exciting new development for the study of dense graphs in the last few years. The approach is now beginning to be adapted to sparse graphs too, and connections are drawn to more traditional ways of forming limits of graphs, such as boundaries – compactifications or metric completions – of infinite graphs.
In a similar development on extremal graph theory’s home turf, we now appear to have the definitive version of the sparse regularity lemma, announced and explained in a major talk at the workshop for the first time.

Another area with striking new results is matroid theory. The structure theory for the finite matroids representable over a given finite field is taking shape now, and the proof that these matroids are well-quasi-ordered as minors appears to be nearing completion. There is now a theory of infinite matroids that admits duality and is based on axiom systems much like the finite matroid axioms; this finally solved a problem of Rado of 1966.

A surprising recent development reflected by the workshop is that, 30 years after its beginnings and more than 10 years after the publication of most of the proof of the graph minor theorem, graph minor structure theory is finally coming of age, being taken up by other researchers at a level comparable to the original papers. Its central technical result, the structure theorem for an excluded minor, has several more mature versions now, partly with new and substantially simpler proofs (which are still difficult but becoming manageable), and applications e.g. in computer science. The same is true for some of the more algorithmic parts of the theory.

In graph connectivity, there are interesting recent attempts to extend to higher connectivity Tutte’s tree-decomposition of a graph into cycles and 3-connected components. The aim is to find, for any fixed integer $k$, a canonical set of nested $k$-separations that shapes the graph into a coarse tree structure made of $(k + 1)$-connected components. This theory started as a tree-structure theorem separating the ends of an infinite graph but is now being applied to separate highly connected finite parts, rather than rays, in a possibly finite graph.

This graph theory week in Oberwolfach was perhaps the liveliest we have ever had. In addition to some excellent main talks it owed most of its spirit to numerous informal workshop organised spontaneously by the participants: we had 12 such gatherings in all, with mostly about 5–10 participants. It was in these workshops that trends such as those mentioned above could really be felt.