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Numerical Techniques for Optimization Problems with PDE Constraints

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Abstract. The development, analysis and implementation of efficient and robust numerical techniques for optimization problems associated with partial differential equations (PDEs) is of utmost importance for the optimal control of processes and the optimal design of structures and systems in modern technology. The successful realization of such techniques invokes a wide variety of challenging mathematical tasks and thus requires the application of adequate methodologies from various mathematical disciplines. During recent years, significant progress has been made in PDE constrained optimization both concerning optimization in function space according to the paradigm 'Optimize first, then discretize' and with regard to the fast and reliable solution of the large-scale problems that typically arise from discretizations of the optimality conditions.

The contributions at this Oberwolfach workshop impressively reflected the progress made in the field. In particular, new insights have been gained in the analysis of optimal control problems for PDEs that have led to vastly improved numerical solution methods. Likewise, breakthroughs have been made in the optimal design of structures and systems, for instance, by the so-called 'all-at-once' approach featuring simultaneous optimization and solution of the underlying PDEs. Finally, new methodologies have been developed for the design of innovative materials and the identification of parameters in multi-scale physical and physiological processes.

Mathematics Subject Classification (2000): 35Rxx, 49-xx, 65Kxx, 90Cxx, 93Cxx.

Introduction by the Organisers

The workshop *Numerical Techniques for Optimization Problems with PDE Constraints*, organised by Matthias Heinkenschloss (Houston), R.H.W Hoppe (Augsburg/Houston), and Volker Schulz (Trier), held January 25th–January 31st, 2009, was the third in series, following two Oberwolfach workshops on the same subject in 2003 and 2006. One of the main objectives of the first two meetings was to bring together leading experts from the fields of optimal control/optimization on one hand and the efficient and reliable numerical solution of PDEs on the other hand in order to encourage and foster new approaches by the exchange of state-of-the-art methods and fresh ideas. The achievement of this goal was well reflected by the 2009 workshop which was attended by almost fifty active researchers from nine countries including a few students and postdoctoral fellows. A total of thirty-one presentations was given at the workshop covering a wide spectrum of issues ranging from the analysis of specific theoretical problems to more algorithmic aspects of computational schemes and various applications in aerodynamics and fluid mechanics as well as life and material sciences.

A particular area of active research, where the adaptation of new insights from optimization and numerical PDEs was extremely beneficial, is the

Numerical solution of control and/or state
constrained optimal control problems for PDEs.

This topic was one of the central themes of the workshop addressed in several talks including the a priori and a posteriori error analysis of numerical schemes (Hinze, Vexler, Weiser), a convergence analysis for the approximate solution of controlled conservation laws (St. Ulbrich), numerically verified bang-bang controls (Tröltzsch), as well as recent progress in mathematical programs with equilibrium constraints (Hintermüller), optimal control of state constrained dynamical systems with ODEs and PDEs (Pesch), robust solution methods via the virtual control approach (Ridzal), and interior-point methods for state constrained problems (Schiela). The important class of semi-smooth Newton methods was studied focusing on independence results (M. Ulbrich) as well as on its application to time-optimal control problems (Kunisch) and to systems of Allen-Cahn variational inequalities (Blank). Further contributions dealt with the efficient solution of PDE control problems with random coefficients (Borzi), the combination of the Hamilton-Jacobi-Bellman approach and Pontryagin's minimum principle (Cristiani), and control problems for elliptic and parabolic PDEs promoting directional sparsity (Griesse).

Another central theme of the workshop was

Optimal Design/Shape and Topology Optimization.

Researchers in these areas reported on the use of game theoretic concepts in multi-objective optimization (Desideri), the state-of-the-art in the analysis and numerics of topology optimization (Leugering), the optimal design of metamaterials (Sigmund), and recent analytical results for shape optimization of the compressible Navier-Stokes equations (Sokolowski). Applications included a PDE approach to optimization/optimal control of high performance buildings (Burns), efficient and fast numerical methods in aerodynamic shape design (Gauger, Schmidt), acoustic optimization of plates and shells (Hardesty), and robust shape optimization in computational fluid dynamics (Schillings). The aspect of shape optimization based on reduced order modeling was covered by a contribution dealing with a combination of domain decomposition and balanced truncation techniques (Antil).

The important topic of

Parameter Identification/Inverse Problems

was the subject of four talks concerned with PDE-based statistical inverse problems in geology (Ghattas), adaptive concepts for parameter identification (Kaltenbacher), reduced order modeling by proper orthogonal decomposition for hydrological inverse problems (Kelley), and parameter estimation for diffusion processes in hippocampal neuron nuclei (Wittum).

The efficient solution of large-scale optimization problems was addressed by a survey on methods based on iterative linear solvers (Gill) and on a priori bounds for the ratio between the cost of an optimization run and a single system simulation (Griewank).