

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Hyperbolic Conservation Laws

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ABSTRACT. The subject of this workshop concerned the analysis and numerical investigation of hyperbolic systems. In particular the interactions between theoretical and numerical contributions were in the focus of this meeting. Many theoretical results have been initiated by numerical methods and experiments and rigorous numerical results are based on fundamental theorems. In particular during this workshop recent results about hyperbolic Monge Ampere systems, elastodynamics, dissipation in systems of conservation laws, Boussinesq, Boltzmann and (error control for the) Navier Stokes equations, mixture of fluids, anelastic and weakly compressible models for atmospherical flows, hydrodynamic limits, wetting and drying for shallow water problems, potential flows, divergence free transport equations, initial boundary Riemann problems, various Discontinuous Galerkin and operator splitting methods, IMEX schemes, entropy stable methods, well balancing schemes, convergence rate for the Glimm scheme and reduced basis methods for conservation laws were discussed.

Mathematics Subject Classification (2000): 35L65, 37K05, 35Q30, 76Nxx, 76Txx.

Introduction by the Organisers

The workshop *Hyperbolic Conservation Laws*, organized by Constantine M. Dafermos (Providence), Dietmar Kröner (Freiburg) and Randall J. LeVeque (Seattle) was held December 7th – 13th, 2008. We had 44 participants from eight different countries. The atmosphere in the Oberwolfach Research Institute was very stimulating and has initiated many fruitful discussions and exchange of ideas. The time schedule was organized as follows: We had three 30-min-lectures in the morning and in the afternoon and for each lecture at least 15 min for discussion. This time was actively used for many questions and remarks from people in the audience. As usual, on Wednesday afternoon we walked to a restaurant in Oberwolfach Kirche.

In the evening on Wednesday D. Serre (Lyon) chaired a problem session for all participants. Many of them spontaneously described some open problems and initiated discussions about some ideas for solving them. The time between the lectures, in the lunch breaks and in the evenings were intensively used for starting new and to continue old cooperations. The participants of the workshop thank the administration of the institute for the possibility for organizing this meeting and the staff for the perfect service.

For this workshop we invited worldwide leading specialist and younger researcher who work on theoretical or numerical questions for hyperbolic conservation laws.

In the field of theoretical contributions the speakers reported on the following results. Existence of solutions for the linear transport equation, if the transport velocity is in L^∞ , was the main subject of Bianchini's contribution. Brenier considered a coupled system consisting of the Monge-Ampère equation and a conservation law that occurs as a "high field limit" of the Vlasov-Poisson system or, alternately, as a simplified model for chemotaxis. The objective of the work of Christoforou was to construct an entropy weak solution of bounded variation for the Riemann problem for systems of conservation laws in one-space dimension via a vanishing viscosity method, for which the viscosity coefficient varies with time. Feldman described in his talk recent results on regular shock reflection for potential flow equation in dimension two. Klingenberg presented a relaxation system for ideal MHD system and derived approximate Riemann solvers with three, five or seven waves, that generalize the HLLC solver for gas dynamics. They satisfy discrete entropy inequalities, and preserve positivity of density and internal energy. T.P. Liu discussed different approaches to study the Boltzmann equations: the probability approach and the CFD approach from continuum mechanics. The purpose of Pan's contribution was to prove the existence of unique global smooth solutions for smooth initial and boundary values for the Boussinesq system. Ruggeri considered different models of a mixture of compressible fluids, and in the case of Euler fluids, the local and global well-posedness of the relative Cauchy problem for smooth solutions. In particular he presented a classical approach of mixture of compressible fluids when each constituent has its own temperature. Serre proved that entropy dissipative viscous extensions of nonlinear systems of conservation laws satisfy under some structure condition for the dissipation tensor the main assumption of Kawashima in his fundamental study of the hyperbolic-parabolic Cauchy problem. The objective of the work of Trivisa was to develop a rigorous mathematical framework based on the principles of continuum physics and to analyze the global in time existence, stability and asymptotic behavior of multicomponent reactive flows. Tzavaras studied the mechanism of shear band formation, in particular the development of a quantitative criterion explaining the onset of instabilities. Westdickenberg proposed a time discretization for the isentropic Euler equations that consists of a sequence of minimization problems and

analyzed the convergence of the approximations towards a measure-valued solution. Riemann problems for the two dimensional compressible Euler system were considered in the talk of Zheng.

The contributions concerning numerical results are the following: Ancona investigated the rate of convergence of an approximate solution of general nonlinear hyperbolic systems constructed by the Glimm scheme for a suitable choice of an equidistributed sampling sequence. A fast scheme for multi-layer shallow water equations, including elliptic regions and drying was considered by Bochut. Feistauer developed a Discontinuous Galerkin scheme for the solution of the initial boundary value problem for the compressible Euler equations which is unconditionally stable and allows the solution of compressible flows for practically all Mach numbers. Kurganov studied convection dominated diffusion problems by an hyperbolic parabolic operator splitting technique. The parabolic solver is partially based on a discretization of the convolution with the Green function formula for the exact solution of the heat equation. Lukáčová-Medviďová proved entropy-stability for a scheme, based on a Roe-type linearization coupled with the multidimensional Finite Volume evolution Galerkin method. Actually this problem was mentioned by Tadmor during the Oberwolfach conference on hyperbolic conservation laws in 2004. A framework within which existing well-balanced schemes may be rederived and reinterpreted, and new ones may be developed more easily was presented in the talk of Noelle. In particular this framework refers to Finite Volume and Discontinuous Galerkin schemes. Makridakis highlighted the main structure of an algorithm for solving the incompressible Navier Stokes equations which permits mesh redistribution with time and the nontrivial characteristics associated with it. Munz introduced a framework based on Riemann problems for diffusion equations to define suitable numerical diffusion fluxes at grid cell interfaces and developed on this basis Discontinuous Galerkin methods. A completely new problem is treated by Ohlberger. He has developed a reduced basis method for parameterized nonlinear conservation laws, discretized by finite volume schemes. Numerical schemes for the BGK model, as a simplified model for the Boltzmann equation, were studied by Puppo.

The contribution by Klein was concerned with the mathematical modeling of atmospheric flows. Most models are characterized by low Mach numbers and Strouhal numbers of order one and lead to zero Mach number incompressible flow models. However, due to the strong pressure and density variations in the vertical direction the incompressible flow models are not sufficient to describe all important features of the flow. The details of this problem were discussed in his contribution.

