

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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## Trends and Developments in Complex Dynamics

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ABSTRACT. We will focus on five aspects of holomorphic dynamics with recent and substantial progress: holomorphic dynamical systems with more than one active or free critical point, dynamics in several complex variables, Thurston's topological characterization of rational functions, dynamics of transcendental functions, and renormalization, rigidity, and a priori bounds. The organising principle of the workshop is to present the state of the art of each aspect, to evaluate the progress achieved and to formulate new questions.

*Mathematics Subject Classification (2000):* 37F15, 37F45, 37F10, 37F30.

### Introduction by the Organisers

Holomorphic dynamics is a thriving field which has experienced tremendous progress over the last 25 years, involving mathematicians such as Douady, Hubbard, McMullen, Milnor, Sullivan, Thurston, and Yoccoz. Holomorphic dynamics is gifted with the tools of conformal and hyperbolic geometry that allow a deep penetration into its nature, with many further applications to real dynamics.

Conformal dynamics is currently in a phase of transition. Previously, most of the successful and deep work has been focused on one-parameter model families: families of quadratic or unimodal polynomials and families of meromorphic transcendental functions with one singular orbit. The theory is particularly advanced in the case of the quadratic family; in fact, a complete measure-theoretic picture of the dynamics was obtained in the real quadratic family (Lyubich, Avila, de Melo, Moreira, and others).

During the past two years, significant breakthroughs have appeared in two important directions: rigidity in polynomial dynamics, and Julia sets of positive

measure. These results answer major long-standing open questions and simultaneously open up new perspectives. Both problems have served as landmarks in the field, and both have a long history going back to Fatou (and more recently, to Ahlfors, Smale and Mostow). The recent results have brought us very close to a full understanding of the model families, and in particular of the real unimodal dynamics.

Time is now ripe for holomorphic dynamics to move on from the study of the special model cases to a general theory of holomorphic dynamics, delivering on the promise that the model cases have impact in greater generality. Holomorphic dynamics is a vast area of research giving impetus to this transition from many directions. Given the time and number of participants we can not cover all of them within one workshop. We have thus chosen to focus on 5 such aspects of holomorphic dynamics with recent and substantial progress.

The organising principle of the workshop is to present the state of the art of each aspect, to evaluate the progress achieved and to formulate questions which may guide the transition.