Mathematisches Forschungsinstitut Oberwolfach

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Affine Algebraic Geometry

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Abstract. Affine geometry deals with algebro-geometric questions of affine varieties that are treated with methods coming from various areas of mathematics like commutative and non-commutative algebra, algebraic, complex analytic and differential geometry, singularity theory and topology. The conference had several main topics. One of them was the famous Jacobian problem, its connections with the Dixmier conjecture and possible algebraic approaches and reductions. A second main theme were questions on Log algebraic varieties, in particular log algebraic surfaces. Thirdly, results on automorphisms of $\mathbb{A}^n$ played a major role, in particular the solution of the Nagata problem, actions of algebraic groups on $\mathbb{A}^n$, Hilbert's $14th$ problem and locally nilpotent derivations. More generally automorphism groups of affine and non-affine varieties, especially in dimension 2 and 3 were treated, and substantial progress on the cancelation problem and embedding problem was presented.

Mathematics Subject Classification (2000): 14Rxx.

Introduction by the Organisers

Affine geometry deals with algebro-geometric questions of affine varieties. In the last decades this area has developed into a systematic discipline with a sizeable international group of researchers, and with methods coming from commutative and non-commutative algebra, algebraic, complex analytic and differential geometry, singularity theory and topology. The meeting was attended by 48 participants, among them the most important senior researchers in this field and many promising young mathematicians.

Especially helpful were the programs for young researchers: the NSF Oberwolfach program, the EU grant and JAMS grant. They allowed to increase the number of young participants considerably.

The conference took place in a very lively atmosphere, made possible by the excellent facilities of the institute. There were 24 talks with a considerable number of lectures given by young researchers at the beginning of their careers. Moreover there were 4 invited lectures that gave an overview over some of the most vivid subfields of Affine Geometry. The program left plenty of time for cooperation and discussion among the participants. We highlight the areas in which new results were presented by the lecturers:

- Jacobian problem, especially its connections with the Dixmier conjecture (Belov-Kanel and Kontsevich) and possible algebraic approaches and reductions.
- Log algebraic varieties; in particular log algebraic surfaces.
- Automorphisms of $\mathbb{A}^n$, in particular tame and wild automorphisms of $\mathbb{A}^3$, Hilbert's $14th$ problem and locally nilpotent derivations.
- Automorphism groups of affine and non-affine varieties, especially in dimension 2 and 3.
- Cancellation problem and embedding problem.

In the first 4 areas there were overview talks given by D. Wright, M. Miyanishi, D. Daigle and Sh. Kaliman. Finally, in a problem session there were presented a number of open questions and problems, which are listed in at the end of this report.