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## Konvexgeometrie

Organised by  
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ABSTRACT. The geometry of convex domains in Euclidean space plays a central role in several branches of mathematics: functional and harmonic analysis, the theory of PDE, linear programming and, increasingly, in the study of other algorithms in computer science. High-dimensional geometry, both the discrete and convex branches of it, has experienced a striking series of developments in the past 5 years. Several examples were presented at this meeting, for example the work of Naor on the non-linear Dvoretzky theorem, that of Paouris on the distribution of the Euclidean norm on a convex domain and the results of Rudelson on the singular values of random matrices.

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### Introduction by the Organisers

The meeting *Konvexgeometrie* organised by K. M. Ball, P. Goodey and P. M. Gruber, was held from December 17 to December 23, 2006. The meeting was attended by some 40 participants working in all areas of convex geometry. The program involved 10 plenary lectures of one hour's duration and about 15 shorter lectures. Some highlights of the program were as follows.

Grigoris Paouris explained his proof that if  $K$  is an isotropic convex body of volume 1 in  $\mathbf{R}^n$  and the random variable  $X$  is uniformly distributed on  $K$ , then for some absolute constant  $C$ ,

$$P(|X| > C\sigma t) \leq e^{-\sqrt{nt}}$$

for all  $t > 1$ , where  $\sigma^2$  is the variance of  $X$ . The estimate is optimal apart from the value of  $C$ . Olivier Guédon then explained joint work with Fleury and Paouris, showing how the method of Paouris yields the central limit theorem for convex bodies, conjectured in 1996 by Ball and recently proved by Klartag.

Assaf Naor described his new results with Manor Mendel which now give a complete picture of the non-linear Dvoretzky Theorem. 20 years ago, Bourgain, Figiel and Milman proved that any  $n$ -point metric space has a subset of size about  $\log n$  which can be embedded in Hilbert space with a constant distortion. In 2003, Bartal, Linial, Mendel and Naor discovered a remarkable threshold phenomenon: that if we allow distortion larger than a factor 2, there are subsets of size a power of  $n$  which are embeddable in Hilbert space. In the recent work the authors determine exactly the correct dependence of the power on the distortion: for each  $\epsilon > 0$  there are subsets of size  $n^{1-\epsilon}$  that are  $O(1/\epsilon)$  embeddable.

Mark Rudelson described his recent estimates for the smallest singular values of almost square random (Gaussian) matrices. Considerably sharpening earlier work of Litvak, Pajor, himself, Tomczak and Vershynin and (in a slightly different direction) Candes and Tao, he established strong bounds for the probability that a random  $N \times n$  matrix maps a point of the unit sphere in  $\mathbf{R}^n$  to a point of small  $\ell_1^N$  norm. This is equivalent to understanding the maximum radius of an almost full-dimensional random section of the cross-polytope. The passage from such estimates to singular numbers uses standard techniques.

Ralph Howard spoke about his recent work (joint with Paul Goodey) on bodies of constant brightness. This follows Howard's recent solution of the problem (dating back to 1926) of whether there exist bodies in 3-space which are of constant width and constant brightness but which are not Euclidean balls.

Richard Gardner gave an account of several new algorithms for the reconstruction of convex bodies from their x-rays in a small number of directions. The new algorithms are robust in that they can accommodate noisy data and are sufficiently simple that convergence proofs are rendered quite straightforward.

Matthias Reitzner gave a survey of the well-developed theory of random polytopes focussing on deviation estimates for the numbers of faces of given dimension. This covers work of himself, Bárány, Vu and others. Ryabogin and Zvavitch described their joint work with Nazarov solving a conjecture of Weil about the characterisation of zonoids.

Semyon Alesker gave an impromptu evening presentation of the recent work of Greg Kuperberg who has given a very short and self-contained proof of the Bourgain-Milman theorem on the product of the volumes of a convex body and its polar. Mahler conjectured that this volume product is minimised by a simplex: the Bourgain-Milman theorem proves this up to a factor of  $(\text{constant})^n$  in  $n$ -dimensions which is what is needed for most applications.