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Mini-Workshop: Dirac Operators in Differential and Noncommutative Geometry

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ABSTRACT.

This mini-workshop brought together mathematicians and physicists working either on classical or on noncommutative differential geometry. Our aim was to show current interests, methods and results within each group and to open the possibility for interaction between the two groups. The first three days were devoted to expository presentations. The remaining two days were devoted to talks on advanced current research problems and results.

Introduction by the Organisers

The mini-workshop, organized by Christian Bär and Andrzej Sitarz, had a very special character. The participating scientists came from two different mathematical communities: differential geometry (working mainly on problems related to the Dirac operator on spin manifolds) and noncommutative geometry (working mainly on concepts of Dirac operators in the framework of spectral geometry as postulated by Alain Connes).

Spin geometry has become an established and very active subfield of Differential Geometry, after Lichnerowicz observed that the Index Theorem yields a topological obstruction against the existence of metrics with positive scalar curvature. The Dirac operator plays a key role in the deep work of Gromov, Lawson, Rosenberg, Stolz and others on manifolds admitting metrics with positive scalar curvature.

The birth of noncommutative geometry offered completely new possibilities for extending some notions of differential geometry into the realm of operator algebras. In Connes' notion of spectral triples the Dirac operator was used to define a (possibly noncommutative) geometry itself rather than being an object derived from a geometry. Since then many interesting examples of noncommutative spaces

and Dirac operators were studied. The equivalence theorem, allowing reconstruction of a spin manifold from a spectral geometry of a commutative algebra was proved only recently and the proof was presented at the workshop.

The aim of the workshop was twofold: to show current interests, methods and results within each group and open the possibility for interaction between two groups. Due to the character of the meeting, first three days were devoted to the expository presentations, when we tried to cover the possibly broadest scope of topics from one subject presented for the participants from the other group. The remaining two days were devoted to talks on advanced current research problems and results, which had closer links to the topics of both groups. During problem sessions in the evenings various open questions were discussed some of which were solved during the week.