

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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**Mini-Workshop:
Logic, Combinatorics and Independence Results**

Organised by

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November 25th – December 2nd, 2006

ABSTRACT. This is a collection of extended abstracts of a mini-workshop “Logic, Combinatorics and Independence results” that took place on November 25 – December 2, 2006 in Oberwolfach. The mini-workshop was attended by 16 researchers from 11 countries and concentrated on one unifying theme: independence results in mathematics. The workshop brought together researchers from different fields (Reverse Mathematics, Proof Theory, WQO theory, Models of Arithmetic, weak arithmetics and Ramsey Theory) with the purpose of building bridges and re-vitalizing existing connections between these fields. The workshop consisted of 16 one-hour talks, two open problem sessions and many informal small group discussions.

Mathematics Subject Classification (2000): 03D20 (recursive functions and relations, subrecursive hierarchies), 03F15 (recursive ordinals and ordinal notations), 03D80 (applications of computability and recursion theory), 03F30 (first-order arithmetic and fragments), 03C62 (models of arithmetic and set theory), 03H15 (nonstandard models of arithmetic), 03E35 (consistency and independence results), 03F03 (proof theory, general), 03F50 (metamathematics of constructive systems), 03F35 (second- and higher-order arithmetic and fragments), 03F45 (provability logics and related algebras), 05D10 (Ramsey theory), 05C05 (trees), 05A18 (partitions of sets), 05D40 (probabilistic methods), 05C83 (graph minors), 11M99 (zeta and L -functions: analytic theory), 11U99 (number theory: connections with logic), 37B10 (dynamical systems: symbolic dynamics).

Introduction by the Organisers

The mini-workshop “Logic, Combinatorics and Independence results” took place on November 25 – December 2, 2006. The workshop was organized by Andrey Bovykin, Lorenzo Carlucci and Andreas Weiermann and attended by 16 participants:

- Lev Beklemishev (Moscow)
- Andrey Bovykin (Liverpool)
- Wilfried Buchholz (Munich)
- Lorenzo Carlucci (Rome)
- Lev Gordeev (Tübingen)
- Henryk Kotlarski (Warszawa)
- Alberto Marcone (Udine)
- Joseph Mileti (Chicago)
- Antonio Montalbán (Wellington)
- Eran Omri (Be'er-Sheva)
- Michael Rathjen (Leeds)
- Sergei Tupailo (Tallinn)
- Stanley Wainer (Leeds)
- Andreas Weiermann (Gent)
- Alan Woods (Crawley)
- Konrad Zdanowski (Warszawa)

There were 16 one-hour talks, two problem sessions and many one-to-one and small group discussions. The workshop brought together researchers specialising in several connected disciplines: Reverse Mathematics (Alberto Marcone, Joseph Mileti, Antonio Montalbán), Proof Theory (Lev Beklemishev, Wilfried Buchholz, Lev Gordeev, Michael Rathjen, Sergei Tupailo, Stanley Wainer, Andreas Weiermann), WQO theory (Alberto Marcone, Antonio Montalbán, Lev Gordeev, Andreas Weiermann), Models of Arithmetic (Andrey Bovykin, Henryk Kotlarski, Konrad Zdanowski, Alan Woods), weak arithmetics (Lev Beklemishev, Alan Woods, Konrad Zdanowski), logical aspects of finite Ramsey Theory (Andrey Bovykin, Lorenzo Carlucci, Henryk Kotlarski, Joseph Mileti, Eran Omri, Andreas Weiermann). However, the central theme of the workshop was first-order unprovable statements and statements of large logical strength. The subject originated in the late 1970s in the work of several mathematicians, most notably Jeff Paris and Harvey Friedman and attracted a large community of researchers at that time. The discoveries of the Paris-Harrington Principle and unprovability of Kruskal’s Theorem provided, forty years after Gödel’s theorems, the first examples of mathematically natural unprovable statements. Since then, many other examples were found in Ramsey Theory, Graph Theory, well-quasi-order theory and other subjects. One of the main objectives of the workshop was to revive research in this area, especially in view of some spectacular recent developments. These developments revealed deep connections between the study of logical strength and several

mathematical disciplines: Analytic Combinatorics, Graph Theory, Tauberian Theory, Number Theory, Dynamical Systems. Another objective of the workshop was to stimulate communication and joint research between researchers from different sub-areas of the subject (Ordinal Analysis, Reverse Mathematics, Models of Arithmetic). All these different areas were represented at the workshop by leading researchers. The workshop was very successful in setting grounds for comparison and interaction of methods from these areas. The two open problem sessions resulted in a list of problems of common interest and in better understanding of possible directions for future research. The talks varied from reports on recent results and proposals of new general approaches to discussions of new strategies to tackle long-standing open problems. Time allowed for free informal discussion and research in small groups that will eventually result in publications.

We would like to thank the Oberwolfach Institute for the wonderful opportunity to hold a meeting there and for providing NSF travel grants to some participants. We also thank all the staff working in the institute for the pleasant experience we all had during our week in Oberwolfach Institute.

