Mathematisches Forschungsinstitut Oberwolfach

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Spatial Random Processes and Statistical Mechanics

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ABSTRACT. The workshop focused on the broad area of spatial random processes and their connection to statistical mechanics. The subjects of interest included random walk in random environment, interacting random walks, polymer models, random fields and spin systems, dynamical problems, metastability as well as problems involving two-dimensional conformal geometry. The workshop brought together many leading researchers in these fields who reported to each other on their recent achievements and exchanged ideas for new problems and potential solutions.

Introduction by the Organisers

The workshop *Spatial Random Processes and Statistical Mechanics* was organized by Kenneth Alexander (USC), Marek Biskup (UCLA), Remco van der Hofstad (Eindhoven) and Vladas Sidoravicius (IMPA Rio de Janeiro). Some fifty participants from four continents attended which included senior researchers as well as mid-carreer and junior scientists, and a few graduate students.

The workshop brought together leading experts in the following fields:

- Interacting random walks and polymers
- Scaling limits in high D and expansion techniques
- Statistical mechanics of interfaces
- Percolation and other systems in 2D
- Random walk in random environment

which represent topics in the wider area of statistical mechanics and interacting random processes where currently substantial progress is being made.

In total, there were 22 talks split between the five days so that each day was focused on one specific topic from the list above. Each day had a long lunch break between 12:30 and 16:30, which made it possible for researchers to discuss the latest developments, exchange new ideas or just enjoy the working conditions of the Institute.

On Wednesday evening, a Special Evening Lecture was delivered by Wendelin Werner on the subject of Conformal Loop Ensembles which was followed by a celebration of the Fields Medal that was awarded to Werner at the ICM Madrid two weeks prior to the meeting. This was considered an important event, as Werner's is the first Fields Medal awarded to probability theory as a field, and to statistical mechanics in particular.

We proceed by a brief overview of the 5 subjects above and a summary of the principal results of those areas that were reported on in the workshop:

Interacting random walks and polymers: The subject of interacting random walks and, in particular, polymer models seems more active than ever. Concerning polymer problems, the present research focuses on detailed understanding of the problem of heteropolymer absorption at an interface and, particularly, the critical line separating the localized and delocalized regimes.

Several polymer talks have been presented at the workshop. Francesco Caravenna talked about pinning of polymers whose monomers interact via a Laplacian interaction. Francis Comets outlined the approach to directed polymer models via multiplicative cascades. Fabio Toninelli presented results on the "rounding" of localization-delocalization transition due to the presence of quenched disorder. Stu Whittington discussed three models of a polymer subject to a force.

Another talk in this category concerned interacting random walk in relation to Bose-Einstein condensation. Here Wolfgang König presented results linking the Gross-Pitaievski approach to Bose-Einstein condensation to certain explicit problems for interacting Brownian motions. These arise as modifications of the standard, Feynman-Kac representation of the interacting Bose gas.

Scaling limits in high D and expansion techniques: High-dimensional statistical mechanics has enjoyed a lot of attention due to the success of the lace-expansion method. Many critical models were analyzed using this technique and mean-field exponents were established above the corresponding upper critical dimension. Several results in this category were presented at the workshop.

Akira Sakai explained his recent breakthrough in applying the lace expansion to the Ising model. The principal idea is to use the random current representation which permits the use of BK-like inequalities. Frank den Hollander's talk focused on the scaling limit of invasion-percolation cluster on a regular tree. Here the surprising fact is that, despite its close relation to the ordinary (critical) percolation, the scaling limits of this object is rather different. Martin Barlow reported on the study of simple random walk on the Incipient Infinite Cluster of directed (spread-out) percolation above 6 dimensions. The main result is that the spectral

dimension of this walk is 4/3, in accord with the so-called Alexander-Orbach conjecture. Mark Holmes discussed expansion techniques for certain self-interacting random walks.

Statistical mechanics of interfaces: Droplets and interfaces have been a very active area of research in probability theory in the last 30 years. At present, the most intensely studied questions are those of detailed geometrical properties of interfaces, including description of their fluctuations and, potentially, scaling limits, and problems of phase separation.

Three talks on these subjects were presented at the workshop: Senya Shlosman outlined a proof of existence of non-translation invariant, Dobrushin-interface states in certain systems with continuum spins. The main novel idea was the use of chessboard estimates, rather than contour formalism which is not quite available in these cases. Ostap Hryniv reported on ongoing work dealing with droplet formation in 2D Ising model. Here the objective is to derive very precise, local-limiting type asymptotic for all quantities. Finally, Dima Ioffe discussed methods for sharp control of random paths—which arise as, e.g., interface lines—possessing a natural regeneration structure. One of the applications of this is the proof of the Ornstein-Zernike behavior which is instrumental for the conclusions mentioned in Ostap Hryniv's talk.

Percolation and other systems in 2D: Two-dimensional statistical mechanics enjoyed tremendous boost in the last 5 years due to Schramm's invention of the Stochastic Loewner Evolution and the ensuing proofs that this process describes interfaces in various models at criticality. One of the fundamental results is that of Smirnov who proved that, for critical percolation on the triangular lattice, the crossing probabilities converge, in the scaling limit, to Cardy's formulas.

The problem with Smirnov's result is that it is extremely special to the (regular) triangular lattice. Recently, a number of people attempted generalizations beyond this setting. At our workshop, Vincent Beffara presented a summary of some of his attempts in this direction and his analysis of various error bounds by comparison with predictions via SLE. On the other hand, Lincoln Chayes described a model of interacting percolation where Cardy's formulas can, after a non-trivial amount of work, be rigorously justified. In his Special Evening Lecture, Wendelin Werner sketched the definitions and properties of the Conformal Loop Ensembles which are the scaling limits of outer boundaries of a class of critical 2D systems.

Another 2D percolation problem was presented in the talk of Rob van den Berg, who explained the relation of the forest-fire models to the problem of self-destructive percolation. One of the main issues is the existence of the forest-fire model beyond a time when an infinite forest first appears; in self-destructive percolation this boils down to an intriguing conjecture on the continuity of the infinite cluster density.

Random walk in random environment: The subject of random walk in random environment is a place of unabating activity, particularly, in the area of quenched

limit laws. The reversible situations are pretty much well understood so the focus is on irreversible problems or complicated geometries.

Four major results of this sort were presented at the workshop: Jean-Dominique Deuschel discussed the irreversible random walks that admit a finite cycle decomposition. While generally non-reversible, this decomposition allows the use of the theory developed for reversible situations. Noam Berger talked about the zero-one law for the speed in sufficiently high dimensions which extends the famous (and old) results of Kalikow. Ofer Zeitouni presented a new proof of diffusivity for RWRE in $d \geq 3$ in the perturbative regime; this result was published 20 years ago by Bricmont and Kupiainen but their proof is, to the present day, deemed quite impenetrable. Finally, as already mentioned, Martin Barlow presented a proof of the Alexander-Orbach conjecture for the simple random walk on the Incipient Infinite Cluster for directed percolation above 6 dimensions.

The atmosphere of the workshop was very friendly, and there was ample discussion, both during and after the lectures. The organizers wish to thank the "Mathematisches Forschungsinstitut Oberwolfach" for their help in the practical organization of the workshop, and in particular for providing a superb environment for this meeting. As the organizing team, we have discussed the possibility of organizing a similar workshop again in about three years. This idea was encouraged by many participants.

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