

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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**Mini-Workshop:  
The Hauptvermutung for High-Dimensional Manifolds**

Organised by  
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August 13th – August 19th, 2006

ABSTRACT. The meeting was devoted to the Kirby-Siebenmann structure theory for high-dimensional topological manifolds and the related disproof of the Hauptvermutung. We found nothing fundamentally wrong with the original work of Kirby and Siebenmann, which is solidly grounded in the literature. Their determination of  $TOP/PL$  depends on Kirby's paper on the Annulus Conjecture and his 'torus trick', and the well-known surgery theoretic classification of homotopy tori.

*Mathematics Subject Classification (2000):* 57A15, 57Q25.

**Introduction by the Organisers**

The Mini-Workshop *The Hauptvermutung for High-Dimensional Manifolds*, organised by Erik Pedersen (Binghamton) and Andrew Ranicki (Edinburgh) was held August 13th–18th, 2006. The meeting was attended by 17 participants, ranging from graduate students to seasoned veterans.

The manifold Hauptvermutung is the conjecture that topological manifolds have a unique combinatorial structure. This conjecture was disproved in 1969 by Kirby and Siebenmann, who used a mixture of geometric and algebraic methods to classify the combinatorial structures on manifolds of dimension  $> 4$ . However, there is some dissatisfaction in the community with the state of the literature on this topic. This has been voiced most forcefully by Novikov, who has written "In particular, the final Kirby-Siebenmann classification of topological multidimensional manifolds therefore is not proved yet in the literature." (<http://front.math.ucdavis.edu/math-ph/0004012>)

At this conference we discussed a number of questions concerning the Hauptvermutung and the structure theory of high-dimensional topological manifolds. These are our conclusions:

We found nothing fundamentally wrong with the original work of Kirby and Siebenmann [4], which is solidly grounded in the literature. Their determination of  $TOP/PL$  depends on Kirby's paper on the Annulus Conjecture and his 'torus trick'. It was noted that Kirby's paper is based on the well-documented work on  $PL$  classification of homotopy tori (Hsiang and Shaneson, Wall) and Sullivan's identification of the  $PL$  normal invariants with  $[-, G/PL]$ , but does not depend on any other work of Sullivan, documented or undocumented. This classification can be reduced to the Farrell Fibring Theorem [1], the calculation of  $\pi_i(G/PL)$  (Kervaire and Milnor [3]), and Wall's non-simply connected surgery theory [7].

There are modern proofs determining the homotopy type of  $TOP/PL$  using either the bounded surgery of Ferry and Pedersen [2] or a modification of the definition of the structure set.

Sullivan's determination of the homotopy type of  $G/PL$ , which is well-documented (for instance, in Madsen and Milgram [5]) is used to determine the homotopy type of  $G/TOP$  and is fundamental to understanding the classification of general topological manifolds.

The 4-fold periodicity of the topological surgery sequence established by Siebenmann [4, p.283] contains a minor error having to do with base points. This is an easily corrected error, and the 4-fold periodicity is true whenever the manifold has a boundary.

The equivalence of the algebraic and topological surgery exact sequence as established by Ranicki [6] was confirmed.

Sullivan's characteristic variety theorem, however it is understood, is not essential for the Kirby-Siebenmann triangulation of manifolds.

The following papers have been commissioned:

- W. Browder, " $PL$  classification of homotopy tori"
- J. Davis, "On the product structure theorem"
- I. Hambleton, " $PL$  classification of homotopy tori"
- M. Kreck, "A proof of Rohlin's theorem"
- E.K. Pedersen, "Determining the homotopy type of  $TOP/PL$  using bounded surgery"
- A. Ranicki, "Siebenmann's periodicity theorem"
- M. Weiss, "Identifying the algebraic and geometric surgery sequences"

The Hauptvermutung website <http://www.maths.ed.ac.uk/~aar/haupt> will record further developments.

#### REFERENCES

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