

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 51/2005

**Mini-Workshop: Dynamics of Cocycles and
One-Dimensional Spectral Theory**

Organised by
David Damanik (Pasadena)
Russell Johnson (Firenze)
Daniel Lenz (Chemnitz)

November 13th – November 19th, 2005

ABSTRACT. Many spectral questions about one-dimensional Schrödinger operators with quasi-periodic potentials can be reduced to dynamical questions about certain quasi-periodic $SL(2, \mathbb{R})$ -valued cocycles. This connection has recently been employed to prove a number of long-standing conjectures. The aim of this mini-workshop was to bring together people from both spectral theory and dynamical systems in order to further develop and exploit the dynamical approach to quasi-periodic Schrödinger operators.

Mathematics Subject Classification (2000): 34C20, 37E45, 47B80.

Introduction by the Organisers

The mini-workshop *Dynamics of Cocycles and One-Dimensional Spectral Theory*, organised by David Damanik (Pasadena), Russell Johnson (Firenze) and Daniel Lenz (Chemnitz), was held November 13–19, 2005.

There have been a number of recent breakthroughs in the spectral theory of one-dimensional Schrödinger operators with quasi-periodic potentials that were accomplished using sophisticated dynamical systems methods; especially by establishing reducibility properties of certain quasi-periodic $SL(2, \mathbb{R})$ -valued cocycles.

The most popular example of a one-dimensional Schrödinger operators with quasi-periodic potential is given by the almost Mathieu operator,

$$[Hu]_n = u_{n+1} + u_{n-1} + 2\lambda \cos(2\pi(n\alpha + \theta))u_n,$$

where $\lambda \neq 0$ and α is irrational. Using the connection with dynamics, it was recently shown for all parameter values that the spectrum of H is a Cantor set of Lebesgue measure $|4 - 4|\lambda||$.

It was the objective of the mini-workshop to bring together experts from both spectral theory and dynamical systems to learn from each other and to further explore potential applications of dynamical systems methods in the context of quasi-periodic Schrödinger operators. Special attention was paid to having many young participants. This was made easy by the fact that currently there are many excellent graduate students and postdocs working on problems located at the interface between the two areas. Consequently, about two thirds of the participants belonged to the age group 35 years or younger.

The talks presented by the participants reflected the current developments in this area. Among other things, there is now an improved understanding of analytic potentials with non-perturbatively small coupling, there are extensions of some results known for analytic potentials to certain classes of non-analytic potentials, while phenomena different from those in the analytic case may occur for potentials of low regularity, and there is improved understanding of the case of Liouville frequencies.

Among the highlights of the discussions outside the talks one could mention the “joining” of independent and closely related work of Bjerklöv and Jäger and the solution by Avila and Damanik of one of the few questions about the almost Mathieu operator that were still left open after the recent advances triggering the mini-workshop.