

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Gemischte und nicht-standard Finite-Elemente-Methoden mit Anwendungen

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ABSTRACT. Mixed and non-conforming finite element methods form a general mathematical framework for the spatial discretisation of partial differential equations, mainly applied to elliptic equations of second order and are becoming increasingly important for the solution of nonlinear problems. These methods are under active discussion in the mathematical and the engineering community and aim of the workshop was to provide a joint forum for the current state of research.

Mathematics Subject Classification (2000): 65N30, 65N15, 65N10, 74S05, 74Bxx, 74Cxx.

Introduction by the Organisers

Mixed finite element methods (MFEM) form a general mathematical framework for the spatial discretisation of partial differential equations, mainly applied to elliptic equations of second order. They become increasingly important for the solution of nonlinear problems. In contrast to standard finite element schemes the mixed finite element discretisation of problems in divergence form, i.e. $f + \operatorname{div} \sigma = 0$ where $\sigma = A(\nabla u)$, $\sigma \in L$ and $u \in H$, allows more flexibility in the design of the discrete approximation spaces contained in L and H , i.e. in the spaces for the direct variables and the Lagrange multipliers.

The workshop focuses on new developments in the field of mixed and non-standard finite element methods. The main points are

- The analysis of mixed FE formulations and of non-conforming methods, including, for instance, enhanced strain and discontinuous Galerkin methods, cf. (F. Armero), (D. Braess), (R. Durán), (K. Garikipati), (P. Monk), (F. Radu), (D. Reddy), (S. Reese), (R. Sacco), (J. Schöberl), (W. Wagner), (R. Winther), (B. Wohlmuth).

- The effective and reliable error estimation of finite element methods as the basis for adaptive techniques and error control, cf. (M. Ainsworth), (P. Houston) and (R. Rannacher).
- Numerical techniques to handle strong and weak discontinuities predicted by the underlying boundary value problem, cf. (P. Steinmann). Cracks and material failure need to be simulated in accordance with the physical experiment, cf. (P. Hansbo), (U. Hoppe), (K. Weinberg).
- For a range of phenomena in structural mechanics the microscale of the material needs to be taken into account to obtain accurate numerical solutions, cf. (T. Arbogast).

The workshop aimed at bridging the gap between the computational engineering community and applied mathematicians and in consequence to unify the scientific language and foster later collaboration. Nonlinear mixed schemes were of particular concern for problems in elasticity and plasticity, but electromagnetics and mathematically related topics were also included.

Mixed finite element methods for elliptic problems are based on a variational description in saddle-point form. Side conditions such as divergence free velocity fields in incompressible fluid dynamics are usually treated in this framework. The appearance of ‘soft’ side conditions is typical for structural mechanics as is the case with nearly incompressible materials or plates and shells with small thickness parameters. We also mention materials which almost satisfy the Kirchhoff condition, i.e. problems with a high but finite shear stiffness. In such cases, which are by no means ‘soft’ from the mathematical point of view, mixed methods lead to a more robust discretisation.

The arising stability conditions and computational techniques cannot be understood fully by intuitive mechanical principles; however, from the mathematician’s point of view their reasoning is natural, clear and insightful.

Mixed and non-standard finite element methods gain increasing prominence in the prevention of locking phenomena. We highlight a topic which is currently actively investigated: the development of stable and efficient plate and shell elements with regard to shear locking, which is more intricate than volume locking. Here it is important to understand how techniques based on heuristic ideas are consistent with more modern mathematical methods.

Availability of fast solvers is decisive for the competitiveness of numerical techniques. For a variety of applications, multigrid methods are crucial for the efficiency of the implementation.

Methods have been proposed which do not appear plausible if one wants to deduce the algorithms directly from the physical model. The advanced methods depend on rigorous error estimators in order to guarantee that the numerical solutions represent the exact solutions of the physical model.

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