Introduction by the Organisers

The Wavelet and Multiscale Methods workshop, organized by Albert Cohen (Paris), Wolfgang Dahmen (Aachen), Ronald A. DeVore (Columbia) and Angela Kunoth (Bonn) was held July 11–17, 2004. A central objective was to bring together leading researchers working in a variety of different areas where multiscale phenomena play an intrinsic role regarding the theoretical understanding of the problems as well as the numerical solution concepts. This meeting was well attended with over 45 participants with a broad geographic representation from all continents. With only very few exceptions, the most prominent representers of the respective fields participated in the workshop.

The main link being multiscale concepts, the overall audience was certainly less homogeneous than in most other workshops. In order to provide a solid ground for discussions and synergies, thirteen of the twenty six talks were longer survey–style lectures by leading experts who had been contacted by the organizers prior to the meeting. The following topics were addressed in the survey–type lectures:

- Adaptive and multiscale methods for integral equations and PDEs;
- problems of high spatial dimension;
- quasi-sparse representation of global operators and hierarchical matrices;
- nonlinear multiresolution techniques and new libraries for image analysis and compression;
- highly localized multiscale representations on the sphere;
- multiscale approaches for inverse problems;
Several adaptive wavelet schemes for fluid flow problems were presented with varying emphasis on analysis and numerical realization. This included combinations of statistical and multiresolution concepts for understanding turbulence. Such approaches were complemented by a “two-scale” point of view where the concept of “residual free bubbles” allows one to model the influence of unresolved scales to the resolved ones, initiating vivid discussions about a possible combination of these approaches. From the perspective of nonlinear approximation, best \( N \)-term approximation in corresponding anisotropic function spaces was discussed, which is another important component in adaptive solution concepts in this context.

Another focal point was the quasi-sparse or data-sparse representation of global operators, for instance, via wavelet bases or through concepts like hierarchical matrices. On one hand, this was shown to lead to new shape-optimization techniques as well as to significant advancements of adaptive wavelet methods for boundary integral equations. On the other hand, hierarchical matrix concepts were seen to make extremely high dimensional problems of a certain separable structure numerically tractable. In fact, problems of high spatial dimension were treated in the context of quantum chemistry centering upon the solution of the Schrödinger equation for many particles as one of the grand challenges of the future. Here compression and sparse grid techniques were again complemented by theoretical investigations of the regularity of eigensolutions which provides the foundation of the potential success of such numerical methods.

Anisotropy is also a key issue in modern image compression techniques. The “wedgelet” concept was presented as a way of extending approximation libraries in combination with image encoding and model selection techniques.

Moreover, the meeting adequately reflected through several talks some very recent advances in the currently highly active area of Mathematical Learning Theory, where new synergies between approximation theory and statistics are just emerging. In particular, wavelet techniques have now led to new universal estimators in a model free regression context where the universality could be achieved through nonlinear approximation concepts.

Regarding the inhomogeneity of the participants’ research fields, in order to maintain a high level of attendance and interest throughout the meeting, the topics of the lectures were deliberately mixed. This created a very open atmosphere and essentially all talks were followed by extensive discussions revealing a number of unforeseen connections between seemingly different subjects. The reaction of the participants during the meeting clearly indicated that the combination of different areas was very well perceived as a source of new ideas and perspectives. New collaborations were initiated and already existing cooperations partly across disciplines were pursued on a visibly intense level.

MSC Classification: