Preface

In the history of von Neumann algebras, originating with the work of F. J. Murray and J. von Neumann in a series of papers in the late 1930s, the biggest breakthrough is probably Tomita–Takesaki theory established around 1970. Following that, many important new developments in the subject were made throughout the 1970s by H. Araki, A. Connes, U. Haagerup, M. Takesaki and others, which are nowadays classical parts of von Neumann algebra theory.

Although a few lengthy treatises on the subject have been published, it still seems difficult for beginners to actually learn the details of the theory from them. In fact, after finishing my graduate studies in 1975, I myself tried earnestly to study Connes' papers – a quite difficult endeavor, since the speed of appearance of Connes' new papers was much faster than my speed of studying them.

This book is based on lectures that I delivered during an intensive course in April 2019 at the Department of Mathematical Analysis of the Budapest University of Technology and Economics in Hungary. The course was designed as a fast-track study of some important and classical parts of von Neumann algebra theory, from which one can gain rigorous background knowledge and can consult the original references and/or suitable, more advanced books for further details. Thus, I selected and lectured on several topics of von Neumann algebras starting from the classical developments in the 1970s. These cover the most fundamental topics in the theory, starting from Tomita–Takesaki theory and extending in other directions concerned with non-commutative integration, which are most useful in applications to mathematical/quantum physics, quantum information theory and so on.

In preparing these lecture notes, I tried to make them as self-contained as possible, assuming only a basic knowledge of functional analysis and measure theory. Except for a few cases, all results are given with complete proofs, which are detailed enough to be understood by a beginner. The references, though, are restricted to what is directly needed within the text, in accordance with the aim and the character of the presentation.

I thought that many people wishing to study von Neumann algebras would find my lecture notes useful, so I posted them on the arXiv in April 2020. Then I received a proposal from EMS Press to publish them in the EMS Series of Lectures in Mathematics. The present book is a corrected and enlarged version of the arXiv preprint [40].

The book consists of 11 chapters and an appendix, each chapter starting with a short introduction. Chapter 1 is an overview of von Neumann algebra theory. Chapter 2 covers Tomita–Takesaki theory. Chapters 3, 6, 7 and 8 are concerned with several fundamental classics – the standard form, Connes' cocycle derivatives, operator-valued weights, Takesaki duality and type III structure theory – based on Tomita–Takesaki theory. Chapters 4, 5 and 9–11 treat several topics more or less concerned with non-commutative integration, such as τ-measurable operators, conditional expectations, relative modular operators and non-commutative L^p-spaces. While von Neumann algebras consist of bounded operators on a Hilbert space, in studying them one needs quite often to deal with unbounded closed (in particular, positive self-adjoint) operators; for instance, the (relative) modular operators are unbounded. Thus, basic facts on positive self-adjoint operators are summarized in Appendix A for the reader's convenience.

In view of the aim of this book, the classification of type III factors and the classification of AFD (= injective) factors due to Connes are not included, but only surveyed, in Sections 1.6 and 1.8 of Chapter 1. Since the 1980s, major advances have been made in von Neumann algebra theory with strong connections to other branches of mathematics; the most brilliant ones are Jones' index theory, Voiculescu's free probability theory and Popa's rigidity theory. While these are beyond the scope of this book, brief surveys on them are given in the final section of Chapter 1 for the interested reader.

Acknowledgments: I would like to thank Milán Mosonyi for inviting me to the aforementioned intensive course on von Neumann algebras in April 2019. In preparing for the lectures, I was able to refresh my knowledge of von Neumann algebras, which I studied over forty years ago. Without his invitation it would not have been possible for me to write these lecture notes. Also, I am grateful to Apostolos Damialis for suggesting that I publish my lecture notes in the EMS Series of Lectures in Mathematics.