

Marquis, Timothée

An introduction to Kac-Moody groups over fields. (English) [Zbl 1405.20003](#)

EMS Textbooks in Mathematics. Zürich: European Mathematical Society (EMS) (ISBN 978-3-03719-187-3/hbk; 978-3-03719-687-8/ebook). xi, 331 p. (2018).

From the Publisher's description: "The aims of this book are twofold:

- to offer an accessible, reader-friendly and self-contained introduction to Kac-Moody algebras and groups;
- to clean the foundations and to provide a unified treatment of the theory.

The book starts with an outline of the classical Lie theory, used to set the scene. Part II provides a self-contained introduction to Kac-Moody algebras. The heart of the book is Part III, which develops an intuitive approach to the construction and fundamental properties of Kac-Moody groups. It is complemented by two appendices, respectively offering introductions to affine group schemes and to the theory of buildings."

Part II treats the Kac-Moody (Lie) algebra \mathfrak{g}_A associated to a generalized Cartan matrix A . We learn about positive roots, real roots, complex roots, the Weyl group, integrable modules, the \mathbb{Z} -form of \mathfrak{g}_A , simplicity. Part III is rather different. The problem is that there are many ways to associate a 'Kac-Moody group' to our Cartan matrix, or to Kac-Moody root data, even over a field of characteristic zero. The author presents about a dozen inequivalent variants, ranging from a discrete group generated by a copy of SL_2 for each positive real root, to the ind-scheme of Mathieu and Rousseau. These ind-schemes go beyond the scope of the book, but the other versions are treated in detail, together with a big diagram of connections. The author is an excellent guide. All constructions are first motivated and then compared. The groups may be discrete or come with a Hausdorff topology. They may be defined in terms of presentations or in terms of integrable representations, or in terms of actions on buildings, or on twin buildings. Several variants have a BN-pair structure. The title of the book refers to fields, but the treatment is not restricted to them.

Reviewer: [Wilberd van der Kallen \(Utrecht\)](#)

MSC:

- [20-02](#) Research monographs (group theory)
- [20G44](#) Kac-Moody groups
- [17B67](#) Kac-Moody (super)algebras; extended affine Lie algebras; toroidal Lie algebras
- [20E42](#) Groups with a BN -pair; buildings
- [20F55](#) Reflection groups; Coxeter groups

Keywords:

real root; Cartan matrix; Kac-Moody algebra; BN-pair; Coxeter complex; integrable representation; Tits functor

Full Text: [DOI](#)