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Regular, quasi-regular and induced representations of infinite-dimensional groups.

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The main aim of this book is to start a systematic development of non-commutative harmonic analysis on infinite dimensional non-locally compact groups. In particular, the book generalizes and studies the notions of regular, quasi-regular and induced representations of infinite dimensional groups and develops an analogue of the orbit method in the case of infinite dimensional “nilpotent” groups.

The book consists of twelve chapters, the bibliography and an index. Chapter 1 contains a lot of classical preliminary material on representations of locally compact groups which makes the book fairly self-contained. Chapter 2 studies irreducibility for the right regular representations of the “nilpotent” groups $B_0^{\mathbb{N}}$ and $B_0^{\mathbb{Z}}$ of infinite (in one or two directions, respectively) upper triangular matrices which have units on the main diagonal and only finitely many non-zero off-diagonal elements. An isomorphism criterion of such representations in case they are irreducible is given.

Chapters 3 and 4 study quasi-regular representations of both $B_0^{\mathbb{N}}$ and $B_0^{\mathbb{Z}}$ and also of the “solvable” group $Bor_0^{\mathbb{N}}$ of infinite in one direction upper triangular matrices. In Chapter 5 one finds a proof of the Dixmier commutation theorem for $B_0^{\mathbb{N}}$ which asserts that the von Neumann algebras generated by the right regular and by the left regular representations of $B_0^{\mathbb{N}}$ coincide. This allows the construction of the operator of modular conjugation and the modular operator used in Tomita-Takesaki theory. Chapter 6 determines when the von Neumann algebra generated by the right (resp. left) regular representations of $B_0^{\mathbb{N}}$ and $B_0^{\mathbb{Z}}$ is a factor and also determines the type of this factor.

Chapter 7 studies induced representations. In particular, the classical construction, due to Mackey, of induced representations of locally compact groups is generalized to infinite dimensional groups. This is achieved by constructing certain quasi-invariant measures on appropriate completions of the original homogeneous space. This, in particular, allows the development of an analogue of the orbit method for the group $B_0^{\mathbb{Z}}$. Chapter 8 provides first steps in describing the duals of the groups $B_0^{\mathbb{N}}$ and $B_0^{\mathbb{Z}}$.

Chapter 9 studies an analogue of the group $B_0^{\mathbb{N}}$ over a finite field. In particular, the problem of irreducibility of its regular and quasi-regular representations is addressed, the Laplace operator is constructed and the commutant is described. Chapter 10 provides irreducibility criteria for Koopman representations of the inductive limit of general linear groups. In Chapter 11 one finds examples of regular representations for non-matrix groups, in particular, the group of diffeomorphisms of the interval, of the circle, the group of local diffeomorphisms of the real line and the group of smooth mappings of a Riemannian manifold into a compact Lie group. Finally, in Chapter 12, for an arbitrary infinite dimensional group G , it is shown how to construct some larger topological group \tilde{G} containing G as a dense subgroup and a G -right-quasi-invariant measure on \tilde{G} . In addition, an analogue of the C^* -algebra for an infinite-dimensional group is discussed.

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