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Elliptic PDEs, measures and capacities. From the Poisson equation to nonlinear Thomas-Fermi problems.

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The present book is a research exposition of old and modern techniques at the interplay between partial differential equations and geometric measure theory. The careful author introduces the reader through main techniques of semilinear elliptic PDEs involving solutions that are not expected to be smooth.

The book contains 22 chapters. Chapter 1 gives an introduction to Laplace and Poisson equations, mean value property and maximum principles. Chapter 2 deals with the Poisson equation $-\Delta u = \mu$ in the sense of distributions. In particular, every superharmonic function satisfies such an equation for some nonnegative Borel measure μ. Various properties of such superharmonic functions are discussed here. Chapter 3 presents the major differences between integrable versus measure data in the study of Dirichlet problems $-\Delta u + g(u) = \mu$ in Ω subject to homogeneous boundary condition $u = 0$ on $\partial\Omega$. The study is carried out in the linear case $g \equiv 0$ first and then in the nonlinear case $g \not\equiv 0$. Chapter 4 deals with variational solutions for the same problem where $\mu \in (W_0^{1,2}(\Omega))'$. Chapter 5 discusses the linear regularity theory for the Dirichlet problem with measure data while Chapter 6 gives an overview on comparison tools available in such a setting: weak and inverse maximum principles, Koto's inequality and its variants, localization principle. Chapter 7 deals with Balayage: weak normal derivative and finite charge up to the boundary. Chapter 8 is concerned with the precise representative using the concept of Lebesgue set. Chapters 9–14 deal with maximal inequalities, Sobolev and Hausdorff capacities, removable singularities, families of solutions respectively strong approximation of measures. Chapters 15–16 present various traces inequalities for Sobolev functions and their connection with capacitary estimates. Chapters 17–18 deal with critical embedding and quasicontinuity. Chapters 19–20 are concerned with nonlinear problems with diffusive measures respectively with extremal solutions. The last two chapters are devoted to absorption problems and Schrödinger operator. Two appendices dealing with Sobolev capacities and Hausdorff measure complement the monograph. Various exercises are proposed througout the book which allow the reader to go deeply into the presented concepts.

The book is a valuable resource for graduate students, researchers and scientists interested in the general field of analysis, particularly in the study of semilinear elliptic equations with nonsmooth data. The whole book stands out by the elegance of the presented results and the clarity of exposition. *Marius Ghergu (Dublin)*