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★**Elliptic PDEs, measures and capacities.**

From the Poisson equations to nonlinear Thomas-Fermi problems.
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With this book the author presents in a self-contained way all the concepts and results needed to study elliptic PDEs with L^1 or measure data either in the linear case

$$\begin{cases} -\Delta u = \mu & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

or in the nonlinear case, e.g.,

$$\begin{cases} -\Delta u + g(u) = \mu & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

where μ is an integrable function or a measure, and Ω is a bounded set.

Indeed the author explains with details and, most of the time, probably in the simplest possible way, basic (and not so basic) results of measure theory, Sobolev spaces and linear elliptic equations useful to anyone interested in elliptic PDEs, along with techniques and results more specific to PDEs with measure data like equations in (1) and (2). Among the subjects covered, let us mention

- Sobolev space theory, including Sobolev embeddings in the critical case, traces, and fractional spaces;
- existence techniques like variational, sub- and super-solutions, and the Perron and balayage methods;
- general properties of solutions (maximum principle, regularity);
- harmonic analysis and geometric measure theory with the study of the Newtonian potential and Sobolev capacities.

One of the core subjects of the book is Sobolev capacities and the author gives a careful study of their relations with Sobolev spaces, the problem of elimination of singularities in elliptic equations and the existence of a solution to the nonlinear problem (2) when g is absorbing with the crucial notion of diffuse measure.

In spite of the fact that these subjects are complex and interrelated, the author manages to present them in a progressive and clear way. Indeed the book is divided into 22 chapters, and each of them is devoted to a particular subject in a way that makes them almost self-contained. The author chooses not to give an exhaustive and up-to-date list of the results concerning each subject but instead concentrates on the most fundamental ones. He not only carefully explains their meaning and importance but also presents more advanced versions usually not covered in textbooks. In each case full detailed proofs are given. Interesting extensions are mentioned. Moreover various exercises of different levels of difficulty appear throughout the text along with their detailed solutions, which makes this book more than suitable for a graduate course. The book contains some elements of measure theory and harmonic analysis that could also be used in an advanced undergraduate course.

In conclusion the author managed to write a book which successfully fulfills two purposes. First it provides an exposition of some of the most fundamental results in harmonic analysis and measure theory that will interest students, teachers and the general mathematician, and it also provides to the researcher all the necessary results and concepts to further study the specialized literature dealing with equations with irregular data. *Nicolas Saintier*