Preface

This book was born in September 2012 during a summer school in Tunisia organized by H. Najar. I would like to thank him very much for this exciting invitation! This book also (strictly) contains my lecture notes for a Master's Degree. At its birth, it was entitled "Little Magnetic Book". There were mainly two reasons for this. Firstly, it was a implied reference to the impressive book by V. Ivrii. Secondly, its former title underlined that its ambition was delimited by a small number of clear and purified intuitions, as the antique *Manual* of Epictetus was.

It is aimed to be a synthesis of recent advances in the spectral theory of the magnetic Schrödinger operator. It is also the opportunity for the author to rethink, simplify, and sometimes correct the ideas of his papers and to present them in a more unified way. Therefore this book can be considered as a catalog of concrete examples of magnetic spectral asymptotics. Since the presentation involves many notions from Spectral Theory, Part 1 provides a concise account of the main concepts and strategies used in the book as well as many examples. Part 2 is devoted to an overview of some known results and to the statement of the main theorems proved in the book. Many points of view are used to describe the discrete spectrum, as well as the eigenfunctions, of the magnetic Laplacian as functions of the (not necessarily) semiclassical parameter: naive powers series expansions, Feshbach–Grushin reductions, WKB constructions, coherent states decompositions, normal forms, etc. It turns out that, despite the simplicity of the expression of the magnetic Laplacian, the influence of the geometry (smooth or not) and of the space variation of the magnetic field often give rise to completely different semiclassical structures, which are governed by effective Hamiltonians reflecting the *magnetic geometry*. In this spirit, two generic examples are presented in Part 4 for the two-dimensional case and three canonical examples involving a boundary in three dimensions are given in Part 5. A feature emphasized here is that many asymptotic problems related to the magnetic Laplacian lead to a dimensional reduction in the spirit of the famous Born–Oppenheimer approximation; accordingly Part 3 is devoted to a simplified theory to get access to the essential ideas. Actually, in the attempt to understand the normal forms of the magnetic Schrödinger operator, one may be tempted to make an analogy with spectral problems coming from the waveguide framework: this is the aim of Part 6.

The reader is warned that this book gravitates towards ideas so that, occasionally, part of the arguments might stay in the shadow to avoid burdensome technical details.

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