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★Spectral theory in Riemannian geometry.

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This book presents an introduction to spectral theory, with an emphasis on properties of the spectrum of the Laplacian on compact Riemannian manifolds or on bounded domains in \mathbb{R}^n .

The author assumes the reader has some knowledge of functional analysis. However, one chapter, or roughly one third of the book, is devoted to functional analysis, particularly that related to the spectral theory of linear operators. This chapter contains many definitions and theorems, some examples, and quite a few proofs. In preparation for the problems he considers later, the author introduces unbounded operators and concentrates on the spectral theory of operators which are compact or which have compact resolvent.

Following a very quick review of Riemannian geometry, the remainder of the text is devoted primarily to spectral properties of the Laplacian on a compact Riemannian manifold or on a bounded set in \mathbb{R}^n . The author introduces the reader to many questions and results related to the spectrum of such operators. Some of these are fairly fundamental, for example, the discreteness of the spectrum and the minimax characterization of eigenvalues, both of which are explored in some detail. Others are results which are less commonly found in textbooks. A few of the topics which are introduced here are the relation of the heat kernel to spectral geometry, examples of isospectral but not isometric manifolds and planar domains, and results regarding prescribing the spectrum on a manifold.

Many, though not all, of the results described in the last third of the book are briefly introduced, with treatment varying in length from a paragraph to a page or two. This book contains approximately thirty exercises, primarily in functional analysis and Riemannian geometry.

There are a number of books devoted to spectral theory. This one differs from others in its wide-ranging introduction to spectral problems in Riemannian geometry.

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