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From Newton to Boltzmann: hard spheres and short-range potentials.

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The monograph deals with “the appearance of irreversibility in gas dynamics”. The physical system under study consists of a large number of particles with short range binary interactions. Two cases are taken into account: hard spheres and particles interacting through smooth, monotonic, compactly supported potentials. The motion of the particles is therefore uniform rectilinear until they remain at a distance greater than a minimum distance. For a large number N of particles, the average behavior can be described by the Liouville equation, which is an evolution equation for the N -particle distribution function. In the Boltzmann-Grad limit, in which N tends to infinity in such a way that the product of N by the minimum distance to the $(d - 1)$ th power (with d the space dimension) is identically equal to 1, the particle dynamics is expected to be described by the Boltzmann equation. This latter is an evolution equation for the kinetic density, which takes into account the effects of both the transport and the binary collisions. An H -theorem holds for this equation, stating that the entropy is a Lyapunov functional for it. The appearance of this irreversibility is a problem on which many researchers have worked starting from Boltzmann. As regards hard spheres, the authors present a self-contained proof of Landford’s theorem, which states that “the distribution function of a system of N particles which are interacting with one another by elastic collisions and are initially independent and smoothly distributed, converges to the solution of the Boltzmann equation” in the Boltzmann-Grad limit. They also present this result for the case of particles interacting through compactly supported potentials, exploiting the work of King.

The main contribution of the authors is a detailed study of trajectories involving recollisions, that is interactions between particles which have already interacted in the past, also indirectly. These trajectories are those which violate independence and are not accounted for in the Boltzmann dynamics. It is proved that in the limit the probability that recollisions do not happen converges to 1. The main limitation is that convergence is proved to hold only on small time intervals.

In particular, the authors introduce the BBGKY hierarchy for the marginals and the pseudomarginals of the N -particle distribution function, respectively in the case of hard spheres and compact potentials, and formally derive the respective limiting Boltzmann hierarchies. After finding uniform a priori estimates for mild solutions of the BBGKY and the Boltzmann hierarchies in suitable uniform Banach spaces, the authors prove convergence of mild solutions of the BBGKY hierarchy to mild solutions of the Boltzmann hierarchy. The convergence is in the sense of convergence of observables, that is averages with respect to the velocity variables. To somehow reduce the case of compact potentials to that of hard spheres, the localized interactions are replaced by pointwise collisions by introducing an artificial boundary. The main differences with respect to the hard sphere case are that pre-collisions and post-collisions states differ also by the microscopic positions of the particles and there is some microscopic shift in time.

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