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★**From Newton to Boltzmann: hard spheres and short-range potentials.**

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In this monograph, the authors are concerned with the rigorous derivation of the (irreversible) classical Boltzmann equation as a limiting case of (reversible) Newtonian molecular dynamics. They provide a self-contained re-visitation of the argument outlined by O. E. Lanford III in his seminal work [in *Dynamical systems, theory and applications (Rencontres, Battelle Res. Inst., Seattle, Wash., 1974)*, 1–111. Lecture Notes in Phys., 38, Springer, Berlin, 1975; MR0479206] for the local-in-time validity of the Boltzmann equation for hard spheres in the Boltzmann-Grad limit of the Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy. Moreover, the authors reconsider the results of F. G. King's thesis [*BBGKY hierarchy for positive potentials*, Univ. California, Berkeley, 1975; MR2625983], which extend Lanford's argument to a Hamiltonian system of particles with short range, repulsive potentials.

In essence, following Lanford and King, the monograph shows that for hard spheres and Hamiltonian systems with short range repulsive interactions, and for short times, statistical states solving the BBGKY hierarchy converge, in the Boltzmann-Grad limit, to solutions of the so-called Boltzmann hierarchy. In particular, independent initial states of the BBGKY hierarchy result, in the limit, into factorized solutions of the Boltzmann hierarchy (propagation of molecular chaos), each factor solving the Boltzmann equation. To prove the convergence, the authors apply Lanford's strategy to compare a suitable series expansion of the BBGKY solutions (in terms of sums of collision trees) with the corresponding one for Boltzmann hierarchy, by checking the term by term convergence.

The book has the merit that it provides nontrivial missing details of Lanford's argument, in particular those concerning the term-wise convergence, incompletely presented in the previous literature (even in the hard-sphere case), and that it clarifies some obscure points and fills some gaps of King's work (mostly related to the proof of the term-wise convergence).

The monograph is divided into four major parts (15 chapters). The first part is a selective, contextual introduction to the main problems and results of the book.

Part II is concerned with the hard-sphere case. It provides a rigorous derivation of the BBGKY hierarchy and a precise statement for the convergence of solutions of BBGKY hierarchy (Theorem 8). This part also includes important consideration on independence, propagation of molecular chaos, and an insight into the strategy of the convergence proof.

Part III deals with short range repulsive potentials. Here, the derivation of the BBGKY hierarchy is more delicate. The convergence theorem (Theorem 11) is formulated for smooth, compactly supported, nondecreasing, repulsive potentials, singular at the origin (Assumption 1.2.1), which satisfy a condition (8.3.1) ensuring that the scattering cross-section is well defined.

The principal contribution of the book emerges from Part IV, which concludes the proofs of the main convergence results (Theorems 8 and 11). The arguments are similar, both for hard spheres and short range potentials, regardless of the nature of the interactions. The key point of the analysis is the proof of the term-wise convergence. This is based on the elimination (control) of re-collisions (which are "bad" events leading to an evolution different from the Boltzmann behavior). To control re-collisions, the authors apply explicitly the properties of the scattering cross-section.

The monograph includes a list of references and a notation index.

After the publication of the printed book, the authors provided an on-line erratum to Chapter 5 at www.ems-ph.org/books/173/Erratum-chapter5.pdf to remove some inconsistencies relative to the functional framework of the book.

The monograph is a good reference for researchers and graduate students interested in the fields of mathematical physics and partial differential equations.

An alternative approach (and also extension to stable short range potentials) has been recently published [M. Pulvirenti, C. Saffirio and S. Simonella, Rev. Math. Phys. **26** (2014), no. 2, 1450001; MR3190204]. *Cecil Pompiliu Grünfeld*