

Preface

The most traditional approach to solving mathematical problems is: start with a difficult problem and simplify it until it becomes easy enough to be solved. However, developing mathematical theories quite often goes in the opposite direction: starting with an “easy” theory one tries to “generalize” it to something more complicated which could hopefully describe a much bigger class of phenomena.

An example of the latter is the development of what is now known as *Khovanov homology*. The *Jones polynomial* is a very elementary classical combinatorial invariant appearing in low dimensional topology. However, as most other known topological invariants, it is not an absolute one (it does not distinguish all knots). Some twelve years ago Mikhail Khovanov developed a very advanced “refinement” of the Jones polynomial which seriously increased the level of theoretical sophistication necessary to be able to define and work with it. Instead of elementary combinatorics and basic algebra, Khovanov’s definition was based on category theory and homological algebra and very soon led to the study of higher categorical structures. This “categorification” of the Jones polynomial created a new direction in topology and attracted a lot of attention from some other parts of mathematics, notably algebra and category theory. Within a few years *categorification* became an intensively studied subject in several mathematical areas. It completely changed the viewpoint on many long-standing problems and led to several spectacular results and applications.

This text is a write-up of the lectures given by the author during the Master Class “Categorification” at Århus University, Denmark, in October 2010. It mostly concentrates on algebraic aspects of the theory, presented in a historical perspective, but also contains several topological applications, in particular, an algebraic (or, more precisely, representation theoretical) approach to categorification of the Jones polynomial mentioned above. The text consists of fifteen sections corresponding to the fifteen one-hour lectures given during the Master Class and fairly accurately describes the content of these lectures. There are some exercises (which were proposed to the participants of the Master Class) collected at the end of the text and a rather extensive list of references. Unfortunately, the time constraints on the Master Class resulted in the fact that several recent developments related to categorification did not make it into the text.

The text is aimed to be an introductory overview of the subject rather than a fully detailed monograph. The emphasis is on definitions, examples and formulations of the results. Most proofs are either briefly outlined or omitted; however, complete proofs can be found by tracking references. It is assumed that the reader is familiar with basics of category theory, representation theory, topology and Lie algebras.