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★**Decorated Teichmüller theory.**
With a foreword by Yuri I. Manin.
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This book gives a self-contained account of the theory of decorated Teichmüller space. It serves as a perfect introduction to the theory for advanced students and researchers, starting with the basics in Chapter 1, to emerging applications of the theory to computational biology in Chapter 6.

For a surface $F_{g,n}$ of genus g with n punctures, the Teichmüller space is the space of marked conformal structures, which, due to the uniformization theorem in dimension 2, can be identified with the space of marked complete hyperbolic structures on $F_{g,n}$. The mapping class group of $F_{g,n}$ naturally acts on the Teichmüller space; the quotient is the moduli space of curves.

For a point in the decorated Teichmüller space, one considers a marked hyperbolic structure together with the choice of a horocycle at each puncture. The definition can be extended to bordered surfaces. (Recently R. Kashaev also described a generalization for closed surfaces.)

The decoration leads to several interesting structures, which are all discussed in the book.

The first is a beautiful parametrization of decorated Teichmüller space by lambda lengths, which were introduced by the author in 1987. Lamdba lengths are associated to a triangulation of the surfaces. Lambda lengths associated to two triangulations related by an elementary change, a so-called flip, are related by the Ptolemy equation. This gives the decorated Teichmüller space a cluster structure. (Similar parametrizations and structures were defined later on higher Teichmüller spaces by V. Fock and A. Goncharov.)

Another result of the decoration is that decorated Teichmüller spaces admit natural cell decompositions, where each cell is labeled by an ideal cell decomposition of the surface $F_{g,n}$, or by isotopy graphs of fat graph spines of $F_{g,n}$. This cell decomposition is a crucial ingredient in several applications. It was used by M. Kontsevich to prove the Witten conjecture, which related the intersection theory of the moduli space to the KdV hierarchy. It also plays a role in the more recent applications to RNA-folding.

The book describes all of this in a largely self-contained manner. The first chapter contains basics on the mapping class group and hyperbolic geometry. Lambda lengths for finite surfaces are introduced in the second chapter, and for infinite surfaces in the third chapter. Chapter 4 describes and proves the cell decomposition of the decorated Teichmüller space. The consequences for the mapping class groupoid, integration on Teichmüller space and Weil-Petersson volumes are developed in Chapter 5. This chapter also discusses compactifications of decorated Teichmüller spaces, which are crucial for further applications of the theory to computing invariants of the mapping class group. Chapter 6 is then devoted to the description of several applications in mathematics (Thompson groups and three-manifold invariants) and theoretical physics (string theory and conformal field theory), as well as computational biology (structure of RNA and proteins). The appendix contains reprints of three articles of the author.

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