Foreword

One can argue that the general idea of a moduli space, viewed in its philosophical dimension, lies at the very heart of modern scientific thinking. A moduli space of whatever nature is an incarnation of the "manifold of possibilities", be it the phase space of Solar System, the Hilbert space of quantum state vectors of a hydrogen atom, or, as in one of the papers of R. Penner and M. Waterman of 1993, the space of secondary structures of an RNA molecule.

At the next stage of scientific reflection, "scientific laws" are introduced further constraining time evolution of a system in its phase space, or specifying a measure on it, its subsets and regions, say "equilibrium points" or "attractors", that are more directly responsible for the explanation of observable phenomena.

And as a final check and a hopeful triumph for a theory, observable phenomena and/or results of controlled experiments are seen to fit (or not to fit ...) their expected patterns in the relevant moduli space, the manifold of possibilities. In pure mathematics, "moduli" of the objects of a given type are associated with the image of a space of parameters on which such objects can depend. Historically early modern examples include Grassmannian spaces of linear subspaces, upper complex half-plane as parameter space for elliptic curves, and brilliant generalizations and innovations due to Riemann. In the second half of the XX century, such thinkers as Alexander Grothendieck and William Thurston contributed their very different visions to the development of this general idea.

The book "Decorated Teichmüller Theory" by R. C. Penner is a beautifully presented survey of some of the most important work of the last two decades dedicated to the moduli of two-dimensional geometric objects, complex and/or Riemannian (metric) surfaces, compact or, more often, satisfying appropriate restrictions on boundaries (finite number of points, or a union of small horocycles etc.). One of the dominating characteristics of Penner's approach is a rich and visually appealing representation of surfaces as embedded in three-dimensional Minkowski space and interacting there with piecewise linear structures such as triangulations, embedded graphs etc., described geometrically in terms of hyperbolic lengths, flatness and combinatorics. One can consider this construction as a descendant of classical Dirichlet–Voronoi methods in the theory of lattices.

Such metric characteristics related to a single surface become then parameters on the moduli space of all such surfaces, and, as Penner's body of research abundantly shows, they are so successfully constructed, that a host of known structures can be very efficiently described in their context. As examples, one can name the Weil–Petersson forms, the Thurston boundary of Teichmüller space, the action of the mapping class group on the decorated Teichmüller space, useful and beautiful cell decompositions, and many more. A version of Penner's cell decomposition was used by Maxim Kontsevich in his spectacular proof of Ed Witten's conjectures on the intersection numbers of moduli spaces.

Returning to the general picture of "manifolds of possible" in science, one must now mention that hyperbolic surfaces embedded in a Minkowski space can be imagined as "world sheets" of quantum strings. This image motivated much interesting research and interaction between mathematicians and string theorists in the physical community inspired by Ed Witten. This interaction, to which Penner contributed a series of important insights and results, underlies one of the many aspects that will attract a student or a researcher to study this book.

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