Preface

The goal of this book is to give some answers to the following general question: *How, and to which extent can we simulate numerically the long time behavior of Hamiltonian partial differential equations* typically arising in many application fields such as quantum mechanics or wave propagations phenomena. Starting from numerical examples, these notes try to provide a relatively complete analysis of the case of the Schrödinger equation in a simple setting (periodic boundary conditions, polynomial nonlinearities) approximated by splitting methods. The objective of this book is to analyze the possible stability and instability phenomena induced by space and time discretizations, and to provide rigorous mathematical explanations for them.

The results presented here originate from many collaborations done in the last 4 years. In particular, Chapter VI is largely inspired by joint works with Arnaud Debussche and Guillaume Dujardin. Chapter VII only exists because of several years of common work with Benoît Grébert. The final results of Chapter VII have been obtained with Rémi Carles. I am happy to warmly thank all of them for their contribution to the present analysis. Many parts of these notes have also taken benefit of many discussions and interactions with several mathematicians before and during my stay at ETH: Dario Bambusi, David Cohen, Ludwig Gauckler, Pierre Germain, Vasile Gradinaru, Ernst Hairer, Ralph Hiptmair, Thomas Kappeler, Peter Kauf, Christian Lubich, Eric Paturel, Katharina Schratz, Christoph Schwab and Julia Schweitzer. My sincere thanks go to all of them.

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