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★**Pangeometry.**

Edited and translated from the French by Athanase Papadopoulos.
Heritage of European Mathematics.

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This is the first complete English translation of Lobachevskii's *Pangeometry* to appear in print. A very brief abridged English edition from an 1867 Italian translation edited by H. P. Manning appeared in 1929. The current work is based on the French edition of 1856. The author has made several minor formal changes to make the reading easier for modern readers and has added an index. The book also contains facsimiles of the 1855 Russian edition and the 1856 French edition, a translation of a French biography of Lobachevskii written as a preface to the 1886 edition of his collected geometric works, and a commentary on the contents of *Pangeometry*. The editor's extensive historical and mathematical annotations to the translation and the biography are a superb addition to the book.

Lobachevskii spent years in a vain attempt to establish Euclid's parallel postulate using the previous postulates and common notions before he replaced Euclid's Fifth Postulate with one that allowed more than one line to pass through a given point not on a given line and be parallel to the given line. In doing so, he established a consistent non-Euclidean geometry. Even though Gauss and Bolyai made significant progress on the problem, Lobachevskii was the first to communicate his results when, in 1826 at the University of Kazan, he gave a lecture on his theory of parallels. An article based on the lecture appeared in installments in the Kazan Messenger in 1829 and 1830. In an attempt to clarify his work, he published in installments, beginning in 1836, "New elements of geometry, with a complete theory of parallel lines" in a Kazan University publication. In an attempt to reach a larger audience he published an article on his geometry in French in Crelle's Journal in 1837. In 1840, Lobachevskii published the first clear account of his work in a German treatise from which Gauss learned of his research. An English translation of the German work, *Geometrical researches on the theory of parallels*, edited by G. B. Halsted, appeared in 1891. A facsimile of that work appears in an appendix to R. Bonola's book [*Non-Euclidean geometry, a critical and historical study of its developments*, translation with additional appendices by H. S. Carslaw, Dover, New York, 1955; [MR0070197](#)]. Just before his death, Lobachevskii's published *Pangeometry*, a comprehensive summary of his work, which contains, using a model-free approach, a survey of the foundations of hyperbolic geometry that serves as a basis for the calculus and its applications in hyperbolic space. W. K. Clifford wrote, "What Vesalius was to Galen, what Copernicus was to Ptolemy, Lobachevskii was to Euclid".

It may seem surprising that it has taken over 150 years for a complete English edition of Lobachevskii's work to appear. The criticism in Russia from M. V. Ostrogradskii and others was harsh and undeserved. Over a century after Lobachevskii's work first appeared, it was still an arduous task to introduce non-Euclidean concepts into the mathematics curriculum in English and American classrooms. In England, members of the Association for the Improvement for Geometrical Teaching (now the Mathematical Association) strove for years to free geometrical teaching from its Euclidean bounds. Excellent accounts of that endeavor can be found in J. L. Richards' *Mathematical*

visions [Academic Press, Boston, MA, 1988; [MR0968441](#)] and J. J. Gray's *Worlds out of nothing* [Springer Undergrad. Math. Ser., Springer, London, 2007; [MR2305283](#)]. Several modern textbooks give detailed accounts of hyperbolic geometry and exhibit the Beltrami-Klein and Poincaré models, including D. C. Kay's *College geometry: a discovery approach* [second edition, Addison-Wesley Longman, Boston, MA, 2001] and M. J. Greenberg's *Euclidean and non-Euclidean geometries* [third edition, Freeman, New York, 1993; [MR1261866](#)].

Nikolaï Ivanovich Lobachevskii was born in Nizhny Novgorod, Russia. He had a problematic childhood. After his father's death, his mother moved to Kazan to be closer to her family. She raised Nikolaï and his two brothers. All three brothers attended the city gymnasium and won scholarships to attend the newly founded Kazan State University, the second-oldest university in Russia. While there, Nikolaï studied mathematics with J. C. M. Bartels, a teacher and close friend of Gauss. He started teaching there at age 18, when he took over for his sick brother. Lobachevskii's association with the school lasted forty years. Piqued by an interest in astronomy, he became an adjunct teacher in the physical and mathematical sciences department, and besides mathematics taught physics and astronomy. He became a full professor in 1822. From 1825 to 1835, he served as the university librarian and curator of the University Museum. While in those positions, he facilitated the cataloging and ordering of books and established a superior geological collection of the rocks and minerals of Russia. He served as the rector of the university from 1827 to 1846 and in that position supervised the construction of several university buildings. In 1846, while keeping his university professorship, he was named assistant to the trustee of the director for the Kazan school district. He served in that position until his death. In spite of being elevated to hereditary nobility by Czar Nikolas in 1827 and marrying well in 1832, Lobachevskii died blind and in poverty in 1856. The Russian and French editions of *Pangeometry* were dictated to an assistant.

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